FORSCHUNGSZENTRUM GENERATIONENVERTRÄGE

der Albert-Ludwigs-Universität Freiburg

DISKUSSIONSBEITRÄGE
DISCUSSION PAPERS

The German statutory pension scheme: Balance Sheet, Cross-Sectional Internal Rates of Return and Implicit Tax Rates

Christoph Metzger

No. 63 – December 2016

The German statutory pension scheme: Balance Sheet, Cross-Sectional Internal Rates of Return and Implicit Tax Rates

Christoph Metzger*

Institute for Public Finance I

Research Center for Generational Contracts

University of Freiburg

December 2016

We present a framework for accounting of the German statutory pension scheme and estimate a balance sheet for the years 2005 until 2012. Extending and applying the methodology proposed by Settergren and Mikula (2005), we estimate the cross-sectional internal rates of return of the German pension scheme over this period. We are able to show that the cross-sectional internal rate of return is mainly financed by increasing contributions and by changing the liabilities not backed by assets. Additionally, our results reveal that from an expenditure perspective, the major part of the internal rate of return is resulting from changing longevity rather than indexation of pension entitlements. Finally, we prove that from a cross-sectional perspective the implicit tax of a pension scheme can mainly be interpreted as an "implicit wealth tax" on pension wealth and subsequently present empirical estimates for these cross-sectional implicit tax rates.

JEL-Classification: E01, H55, H83, H87

Keywords: accounting of pension schemes, balance sheet, internal rate of return, implicit tax, fiscal sustainability

---

*Institute for Public Finance I, Albert-Ludwigs-Universität Freiburg, Bertoldstr. 17, 79085 Freiburg, Germany (Tel: +49-761-203-9236, christoph.metzger@vwl.uni-freiburg.de). Thanks go to Stefan Moog, Christoph Freudenberg and Natalie Laub for a lot of discussions, review and advice as well as to two anonymous referees for valuable comments.

1. Introduction

Unfunded Social security pension schemes provide a large fraction of income during retirement.[1] Hence, unfunded pension entitlements[2] of these schemes simultaneously represent a significant part of household assets and constitute implicit government liabilities or debt.[3] Being mostly financed on pay-as-you-go (PAYGO) basis, these unfunded entitlements must be met by future contributions. Thus, long-term financing needs of the pension scheme are non-transparent and may be shifted to the future at the expense of future contributors.

There are two possibilities in order to make these intergenerational redistribution visible. First, one may run a simulation of the pension scheme using various assumptions for key parameters, concerning for example the number of future contributors and evolution of mortality. A second possibility is to apply the balancing mechanisms of funded pension schemes to unfunded ones. The Swedish notional-defined contribution scheme is in principle based on the accounting framework for funded pension schemes and also in the US and Canada balance sheets are estimated in addition to long-run projections using a similar methodology.[4]

Balancing of unfunded pension schemes brings along several advantages. It increases transparency regarding the long-term financing situation of the pension scheme and for example directly reveals the long run impact of reforms. This is especially true if balance sheets are compiled on a regular basis, making changes in balancing items visible over time.

Additionally, if balancing is done regularly using actual cross-sectional data, there is no need to make wide ranging assumptions, as changes in underlying parameters are automatically included only if they actually occur. Thus, there is less space for manipulation with respect to key assumptions for projections.

[1] See e.g. OECD (2013), p.71.
[2] The term pension wealth will be used synonymously.
[3] See Kaier and Müller (2015).
[4] See Board of Trustees (2016) for the US and Government of Canada (2015).

Applying the balancing framework presented by Settergren and Mikula (2005), we first estimate and present a balance sheet of the German statutory pension scheme for the years 2005 until 2012. Similar work has already been carried out for other countries (for example by Boado-Penas et al. (2008) for the Spanish pension scheme and by Billig and Menard (2013) for the Canada Pension Plan). However, to our knowledge, we are the first to estimate a balance sheet for the German pension insurance and to disentangle the evolution of the balancing items in order to estimate the cross-sectional internal rate of return (IRR) of the pension scheme over the observed period. In this paper, we extend the work of Settergren and Mikula (2005), by including public subsidies and accounting for the existence of an unfunded part of liabilities. In a second step, we estimate that part of the IRR arising from decreasing mortality and thus increasing longevity, which might be interpreted as the insurance value against longevity.

The existence of a compulsory statutory pension scheme may induce distortions with respect to individual or household life-cycle planning, whether fully funded or financed on a pay-as-you-go basis. These distortions may result either from fixed contributions being different from what individuals would have chosen in the absence of a pension scheme and/or if the rate of return on savings to the pension scheme differs from the market rate of return, constituting an implicit tax of the pension scheme. While this implicit tax has been widely analyzed in a life-cycle context (for example Luedecke (1988), Sinn (2000), Beckmann (2000) and Fenge et al. (2000), to name just the most important), we have a further look at the implicit tax of a pension scheme from a cross-sectional perspective, extending the work of Fenge et al. (2002) and Beckmann (2000). For this purpose, we first present and discuss the cross-sectional properties of the implicit tax of the pension scheme in a simple theoretical framework and subsequently estimate the implicit cross-sectional tax rate of the German pension scheme for the years 2006 to 2012 using our previous cross-sectional estimates of the internal rate of return (IRR).

2. Methodology[5]

The periodical budget balance of a pension scheme financed on a pay-as-you-go basis is based on a flow concept and states in its simplest form that contributions have to equal pension payments.[6] Thus, already accrued pension entitlements even for the following year are neglected. In order to cope with these shortcomings usually accompanying long-term projections of contribution and pension flows are carried out, considering also new entrants into the system (so-called open group approach). For an infinite horizon there already exists an accounting framework with respect to sustainability, namely Generational Accounting.[7] Generational Accounting itself relies on long-term projections using assumptions about future migration flows and fertility rates. With respect to the considered time horizon, the balancing approach used here lies in between the pure periodic budget balance and the infinite horizon of Generational Accounting. It takes into account all pension entitlements by pensioners and contributors accrued up-to-date but simultaneously neglects future entries into the pension scheme (so-called closed group approach). Although liabilities estimation is based on a closed group approach, contributions of new entrants are considered to some extent under the assumption of a constant population structure and thus constant migration and fertility patterns through the "Contribution Asset" (further outlined in section 2.3.2.).

The definition of sustainability being answered with the "Swedish" balancing methodology used here is thus different from that of Generational Accounting. The latter assesses sustainability of the pension scheme in the very long run given the underlying assumptions about future development of e.g. migration flows and fertility rates. By contrast, the here used "Swedish balancing" measures sustainability of the pension scheme from a different

[5] The following chapter relies widely on Metzger (2016).
[6] If there exists a kind of buffer fund, a potential financing gap has to be balanced by a change in this buffer fund.
[7] For an overview on the methodology see for example Auerbach et al. (1991) and Auerbach et al. (1994).

perspective, namely under a steady state assumption with respect to base year characteristics. Hence, the sustainability measure to be answered is whether the pension scheme itself is sustainable if population structure was to stay constant, without considering future changes in migration and/or fertility. It resembles to a cross-sectional snapshot considering the current situation and therefore yields an informative measure for changes over time, taking into account actually occurred demographic and economic developments. Generally, the Swedish balancing methodology can be regarded as an application of double-entry bookkeeping, standard for private or occupational funded pension schemes, to pay-as-you-go financed social security pension schemes. In line with standard double-entry bookkeeping, balancing of a pension scheme requires consideration of all types of both liabilities and assets. In the following sections first the concept of balancing a PAYGO pension scheme is developed in general. Afterwards the balancing positions are outlined in more detail.

2.1. Budget balance

With the aim of assessing sustainability or solvency, Balancing a PAYGO pension system should follow the same basic accounting principles as balancing a funded pension scheme. The appropriate budget constraint states that total assets have to equal total liabilities:

Eq. 1 $$\boldsymbol{Liabilities - Assets = 0}$$

The liabilities of a pension system, to start with the more intuitive part, consist of the pension liabilities to all individuals currently entitled to a pension payment in the future, to current pensioners as well as to current contributors. In the further context the term pension liabilities ($\boldsymbol{PL_t}$) is used in line with Settergren and Mikula (2005) and describes pension liabilities net of contributions.[8]

[8] A similar concept of liabilities is also applied by Billig and Menard (2013), Boado-Penas et al. (2008) and Vidal-Melia and Boado-Penas (2013).

With respect to assets, several types of assets can be distinguished. Most pension schemes have at least some financial reserves or a small buffer fund (BF_t) in order to cope with short-period deficits.[9] However, for a PAYGO pension system the most relevant source for financing existing pension liabilities are the contributions of current and future generations. Relying on future contributions, or the "contribution asset" (CA_t) as it is referred to by Settergren and Mikula (2005), is therefore an inherent feature of all pension schemes based on pay-as-you-go financing.[10] We extend the more general framework of Settergren and Mikula (2005) by the inclusion of public subsidies, hereafter called public contribution asset (PCA_t). Many public pension schemes rely to some extent on public financing via taxes either as a general subsidy or as targeted contributions in order to finance specific subsidies or benefits e.g. with respect to child or elderly care.

However, if the overall budget constraint does not add up to zero, the remaining unfunded (overfunded) part of the liabilities can also be interpreted as uncovered liabilities (UL_t) at the expense of future (current) generations.[11] Combining these balance positions leads to Equation 2.

Eq. 2 $$PL_t = BF_t + CA_t + PCA_t + UL_t$$

2.2. Liabilities

The liabilities of a pension scheme are based on the promise to pay pensions in the future to the members of the scheme, current pensioners as well as current actively or passively insured

[9] For an overview see OECD (2013), p.195.
[10] The definition and features of this „contribution asset“ are discussed in more detail in section 2.3.2.
[11] Thereby, unintended intergenerational redistribution corresponds to the definition of "accumulated deficit" used by Boado-Penas et al. (2008). Actually, both definitions measure the same amount, namely the uncovered liabilities. The term "accumulated deficit" emphasizes the fact that these deficits have implicitly already been accumulated whereas our focus is on the burden imposed on future generations through unsustainable financing of the pension system.

individuals. Thus, each liability simultaneously represents an individual's asset or entitlement. The sum of all entitlements to individuals is therefore equal to the total (actuarial) liabilities of the pension scheme. These entitlements comprise of old-age, disability as well as survivor's pension entitlements.

In the following we will refer to the concept of accrued-to-date liabilities (ADL). This concept includes the present value of all pension entitlements accrued up to the respective base year; no future accrual of pension entitlements due to contributions or additional service years is taken into account.[12] Hence, it measures the total amount of pension liabilities accrued up-to-date to be paid out if the pension scheme is terminated today.

In general, our estimation of accrued-to-date liabilities is based on Eq. 3 for all types of pensions (old-age, disability and survivor's pensions), although here it is shown exemplary for old-age pensions. As it is shown, pension entitlements $\boldsymbol{E}_{x,g,b}^{oldage}$ in base year *b* for a person of gender *g* and age *x* depend on four factors. The first factor represents the annual pension benefit accrued-to date $\boldsymbol{B}_{s,g,b}^{oldage}$ in base year *b*, to be paid out at future age *s* in future year *f*, with *D* denoting the maximum considered age. Here, pension indexation is already included in this future pension benefit.[13] For current pensioners $\boldsymbol{B}_{s,g,b}^{oldage}$ amounts to their paid out pension, as it is by definition fully accrued. For individuals not yet receiving a pension, this accrued benefit equals the pension rights already accrued in their pension account to be paid in the future.

The probability of an *x* year old person of gender *g* in the base year to survive until the future age *s* is denoted by $\boldsymbol{p}_{s,g,b}^{accum}$. Corresponding to the base year balancing perspective, the survival

[12] The concept of ADL used here, is equal to the term "accumulated benefit obligation (ABO)" used in accounting as well as the term actuarial (accrued) liability. For a more detailed description of the different distinctions of liabilities in the context of unfunded pension systems see Holzmann et al. (2004), Eurostat (2011) or Kaier and Müller (2015).

[13] A special feature of the German pension scheme is the fact that pension indexation takes place in same way for paid out pensions as well as for accrued entitlements.

probabilities of the base year are used here. The third factor for the entitlement calculations is called the occurrence probability $\boldsymbol{p_{s,g,b}^{occur}}$. With respect to the occurrence probability it is useful to distinguish between pensioners already receiving a pension and other entitlements to paid out in the future. By definition the occurrence probability for pensions paid equals one as the pension granting event has already occurred. For entitlements not paid out yet, these probabilities differ from one depending on the type of pension. The occurrence probability for an old-age pension to be paid out in the future equals zero before the retirement age and one afterwards.

Eq. 3
$$\boldsymbol{E_{x,g,b}^{oldage} = \sum_{s=x+1}^{D} [B_{s,g,f}^{oldage} * p_{s,g,b}^{occur} * p_{s,g,b}^{accum} * (1+d)^{x-s}]}$$

By contrast, the occurrence probability for receiving a disability pension depending on age and gender is positive before the retirement age and zero afterwards. The occurrence probability for a survivor's pension not paid out yet depends on the probability of being married, the survival probability of the receiving spouse as well as on the probability that the spouse or parent, from whose entitlement the survivor's pension is deducted, is dead. Finally, *d* represents the discount rate for future payments.[14]

2.3. Assets

Having outlined the concept and calculation of the liabilities entering the balance sheet, the following section will present and discuss the calculation of the assets usually present in a pay-as-you-go pension scheme.

2.3.1. Buffer fund

First of all, financial reserves, e.g. buffer funds, are obviously assets of the pension scheme, with data on their amount being usually available through official statistics. The size of the

[14] The choice of the appropriate discount rate will be discussed in more detail in section 3.1.

buffer fund of a pension scheme can vary widely from about two times the monthly expenditures in Germany[15] to over four times the yearly pension disbursements in Sweden[16]. In the case of Switzerland the buffer fund of the old-age pension system (AHV-Fonds) is operating financial reserves of about 1.20 times the yearly pension expenditures.

From an accounting perspective the buffer fund should enter the balancing sheet in nominal terms, as the rate of return of it equals the appropriate discount rate, the rate of return on financial assets.

2.3.2. Contribution asset

Although it is obvious that some kind of an implicit liability to future generations is existent in every pension scheme financed on a PAYGO basis, it is nevertheless difficult to estimate the exact amount of this “intended” intergenerational redistribution or contribution asset (*CA*).[17] While some authors have proposed to use the sum of implicit taxes or the so-called “hidden asset” as a measure of the implicit intergenerational redistribution,[18] we will stick to the concept of a “contribution asset” as outlined by Settergren and Mikula (2005) for one reason. The existence and the size of implicit taxes depend on the difference between the internal rate of return of the pension system and the return of an alternative investment, the interest rate of the financial market.[19] As we are interested in the sustainability of the pension scheme itself, the more appropriate measure of the implicit intergenerational redistribution seems to be the system-inherent contribution asset as it abstracts from any investment comparisons and therefore from other factors determining the financial market interest rate.[20]

[15] See Deutsche Rentenversicherung (2015), p.26.
[16] See Swedish Pension Agency (2014).
[17] Lee (1994) uses a similar concept in order to measure the claim on future contributions but calls it transfer wealth.
[18] See for example Sinn (2000), Fenge and Werding (2003) and Vidal-Melia and Boado-Penas (2013).
[19] See Vidal-Melia and Boado-Penas (2013).
[20] See Boado-Penas et al. (2008), p.94.

The concept of the contribution asset refers to the sum of contributions it takes, until all liabilities within the pension system are turned over once given stable population, as well as income and pension replacement rate patterns (steady state). As a starting point one has to ask, how long does it take on average for one euro of contributions until it is paid out as a pension benefit? This equals the duration until all liabilities are on average turned over once. In order to obtain this turnover duration $\boldsymbol{TD_t}$ the money weighted average age of contributors $\boldsymbol{A_t^c}$ is deducted from the money weighted average age of retirees $\boldsymbol{A_t^r}$.

Eq. 4 $$\boldsymbol{TD_t = A_t^r - A_t^c}$$

The contribution asset therefore equals the turnover duration times the sum of contributions of the base year ($\boldsymbol{C_t}$).[21]

Eq. 5 $$\boldsymbol{CA_t = TD_t * C_t}$$

The calculation of the contribution asset is generally based on cross-sectional population and pension data due to the steady state assumption. It therefore relies on the actually observed situation and does not take into account possible but uncertain developments in the future. However, if calculated on a regularly basis, it is a repeated measure of the expected inherent redistribution and therefore an important indicator for the evolution of the long-term contribution base of the pension system. The turnover duration and thus the contribution asset only measures the implicit assets of the pension scheme correctly in steady state with stable mortality, income and replacement rate patterns.[22] Yet, if the time interval between new estimates of the contribution asset is sufficient small (one year or even three years), each estimate between may be interpreted as a transitory steady state.

[21] For a more detailed derivation and discussion of the contribution asset see Settergren and Mikula (2005) and especially Vidal-Melia and Boado-Penas (2013).
[22] For a more detailed discussion compare Settergren and Mikula (2005), p.121.

2.3.3. Public contribution asset

In most countries additional subsidies from general tax revenue also play a significant role in financing the pension system. These public subsidies may be connected to the total amount of pensions paid or may be nominally fixed and perhaps also indexed to wages, prices or tax revenue. While calculating the present value of public subsidies connected to pension expenses is fairly simple, the consideration of other public subsidies is not that intuitive. For example, in Germany there exist three different public subsidies: The "General federal subsidy" (Allgemeiner Bundeszuschuss) as the largest part was once nominally fixed and is annually indexed via gross wage growth and changes in the contribution rate. Thus, it is effectively indexed along growth of average contributions. The "additional federal subsidy" (Zusätzlicher Bundeszuschuss) is meant to compensate for benefits not covered by contributions and is indexed along the growth in sales tax revenue. The last block of public subsidies, the "increment amount" (Erhöhungsbeitrag zum zusätzlichen Bundeszuschuss) follows the evolution of average gross wages.

The difficulty with these pension unrelated subsidies is to consider the fraction relevant for financing the existing pension liabilities of the base year. As these liabilities are on average completely turned over within the length of the turnover duration, we only consider the pension unrelated subsidies to be paid for the existing base year liabilities and thus only until these liabilities are completely turned over. Accordingly, we use the calculation methodology of the contribution asset in order to estimate the asset stemming from these public subsidies unrelated to the pension expenses. The resulting public contribution asset ($\boldsymbol{PCA_t}$) consists of the pension unrelated subsidies ($\boldsymbol{S_t}$) times the turnover duration.

Eq. 6 $$\boldsymbol{PCA_t = TD_t * S_t}$$

Using the same calculation methodology as for the contribution asset requires implicit discount rate for future public subsidies being equal to average wage growth.[23] With respect to the steady-state fiction with a constant contribution rate, this assumption is reasonable for the subsidies being indexed with average gross wage growth or growth in average contributions ("general federal subsidy" and "increment amount"). With constant population and consumption patterns, one could probably also argue that sales tax revenue follows average wage growth and thus also the "additional federal subsidy" is indexed in line with average wage growth.

3. Empirical results

Having outlined the calculation methodology for all balancing items, the following section presents first the remaining assumptions needed. Subsequently the empirical estimates for the balance sheet of the German statutory pension insurance for the years 2005 until 2012 are presented. Based on the changes of the balance positions the cross-sectional internal rate of return of the pension scheme for these years is estimated in detail and decomposed with respect to the underlying changes. Additionally the features of the implicit tax rate on contributions and entitlements to the pension scheme are presented and discussed from a cross-sectional perspective using a simple model. Following, the empirical estimates for the implicit tax rate of the German pension insurance are presented. All calculations are based on official statistics of the German pension insurance on insured persons, their average entitlements, on the number of pensioners, the respective pension amounts as well as on aggregate financing flows.

[23] Compare Vidal-Melia and Boado-Penas (2013), p.1307.

3.1. Assumptions

The main assumption within this balancing framework is the assumption of a steady-state. This results in considering cross-sectional facts of the base year only without relying on projections. New information for example about increases in longevity are incorporated in the balance sheet only if they have actually occurred. Thus, the balance sheet takes into account only observed increases in longevity and prevents manipulation or biases with respect to projection assumptions.[24] In order to be consistent with this assumption, one has to use base year periodical life tables in order to estimate the liabilities. The age and gender specific death rates for each year are taken from Eurostat. The age- and gender specific probabilities for newly entering the disability pension scheme are calculated for each year using the number of new entrants divided by the number of insured individuals of that age and gender.

A crucial question within the calculation of pension liabilities in present value terms regards the choice of the appropriate discount rate. As Aaron (1966) has already pointed out, the implicit rate of return of a PAYGO pension system equals the growth of the wage bill or the sum of wage and working population growth.[25] If future changes in the working population are neglected given the steady-state assumption, which is also applied in the estimation of the contribution asset, the appropriate internal rate of return of the pension system equals wage growth.[26] For the estimation of future pension benefits we assume an indexation in line with gross wage growth, which is the case in Germany under a steady-state perspective. Actually pensions are indexed according to modified wage growth taking negatively into account increases in the contribution rate. Additionally a sustainability factor connects this modified

[24] See also Boado-Penas et al. (2008), p.103.
[25] As Settergren and Mikula (2005) point out, this is only true if increases in longevity are neglected.
[26] The growth in the contributory base is also proposed or used as discount factor for pay-as-you-go financed liabilities by Settergren and Mikula (2005), Boado-Penas et al. (2008) and Billig and Menard (2013). While in our context we choose the rate of wage growth as the appropriate discount rate for the calculation of the accrued-to-date pension liabilities, a real discount rate of 3 percent reflecting the European average ten-year government bond yields is recommended by Eurostat (2011) for estimation of ADL.

wage growth indexation approximately to the development of the ratio of pensioners to contributors.[27] However, we do not take into account future implications of the sustainability factor due to our base year balancing perspective with the underlying steady-state assumption. With the gross wage growth as the appropriate discount rate, our estimates are completely independent of the exact choice of the discount rate as pensions and entitlements are indexed with the same rate.

3.2. Balance sheet

Based on the methodology and assumptions outlined in the preceding sections, the estimated balance sheet for the German pension scheme is presented in the following. The balance sheet is estimated for the years 2005 until 2012.

With respect to liabilities, old-age pensions represent the biggest share of liabilities with about 8,000 to 8,700 billion Euro. Liabilities for disability and survivor's pensions amount both to about 1,200 billion Euro in the first years. While liabilities for disability pensions decrease later, liabilities for survivor's pensions increase probably due to population aging. The older the working population, the smaller is the cumulated probability of receiving a disability pension before the regular retirement age.

Real financial assets are nearly ignorable for the German social security pension scheme as they reach their maximum of 0.33 percent of overall liabilities in 2012. By definition, the major part of assets in a pay-as-you-go financed pension scheme are future contributions hereby measured by the contribution asset. But also the public contribution asset amounts to between 26 and 29 percent of all assets emphasizing the importance of public subsidies to the German pension

[27] Measured as average pensioners and average contributors. For more details and a projection of the pension indexation compared to wage growth see for example Geyer and Steiner (2014).

scheme. Thus, it is necessary to incorporate public subsidies if estimating a balance sheet for a pension scheme depending partly on public subsidies from tax revenue.

Table 1: Balance sheet of the German pension insurance in Bn. Euro

		2005	2006	2007	2008	2009	2010	2011	2012
Liabilites		**7,834**	**7,971**	**8,006**	**8,020**	**8,144**	**8,417**	**8,619**	**8,732**
Old-age pensions	(current pensioners)	5,439	5,571	5,641	5,699	5,794	6,031	6,222	6,366
Disability pensions	(current active)	1,160	1,153	1,124	1,074	1,095	1,123	1,089	1,059
Survivor's pensions	(current active)	1,235	1,247	1,241	1,246	1,254	1,263	1,308	1,307
Assets		**6,885**	**7,213**	**7,136**	**7,312**	**7,367**	**7,549**	**7,683**	**7,861**
Funds	(end of year)	2	10	11	16	16	19	24	29
Turnover duration (TD)	(end of year)	29.3	29.2	29.5	29.5	29.4	29.5	29.5	29.7
Public contribution asset (PCA)	(Federal contributions x Turnover duration)	1,941	1,934	1,974	1,986	2,007	2,059	2,054	2,086
Contribution asset (CA)	(Turnover duration x contributions)	4,942	5,269	5,150	5,310	5,345	5,472	5,606	5,745
Accumulated deficit/ Uncovered liabilities		949.0	758.1	870.7	708.2	776.7	867.8	935.8	871.1
Funding ratio/Solvency ratio	(= Assets /(net-) liabilities)	**0.879**	**0.905**	**0.891**	**0.912**	**0.905**	**0.897**	**0.891**	**0.900**

Source: Own calculations based on official statistics from the German pension insurance (DRV).

Taking the "Swedish perspective", a fully balanced pension scheme requires assets being equal to liabilities. If not, the balance ratio (ratio of assets to liabilities) is lower than one, indicating a shift of uncovered liabilities to current and future contributors. As is evident from table 1, the balance ratio of the German pension scheme is fluctuating around 0.9, leaving about 10 percent of all liabilities unbalanced by corresponding assets.[28] Over the observed time horizon this ratio has been pretty stable, ranging from 0.878 to 0.911.

The German sustainability factor links general wage indexation of pensions additionally to the evolution of the ratio of average pensioners to average contributors. Thus, the overall situation of the pay-as-you-go financing is taken explicitly into account. Although the sustainability factor differs from the Swedish balancing ratio with respect to the consideration of the turnover duration, it seems to keep the pension scheme balanced in a "Swedish sense".

3.3. Internal Rate of Return

In this section we will present estimates for the cross-sectional internal rate of return (IRR) of the German statutory pension scheme. It has to be noted that we estimate the average IRR over all members of the pension scheme. Thus individual rates of return may significantly differ from our estimates due to differences in legal rules or for example in longevity etc.

The following calculation methodology mainly relies on Settergren and Mikula (2005). However, it does differ in two crucial points. The methodology outlined by Settergren and Mikula (2005) is explicitly proposed for accounting of a balanced pension scheme. By contrast, we extend this framework to additionally incorporating the uncovered liabilities, thus making it also applicable for non-balanced pension systems. If for example policymakers enact a reform

[28] There exist already some other studies applying the Swedish balancing methodology on pay-as-you-go financed pension schemes. Using cohort mortality Billig and Menard (2013) estimate a balance ratio of 1.05 for the Canada Pension Plan and Metzger (2016) a balance ratio of 0.7 for the Swiss old-age pension scheme (without disability pensions). By contrast, Boado-Penas et al. (2008) use base year mortality and estimate a balance ratio or what they call a ratio of (in)solvency between 0.67 and 0.74 for the Spanish pension scheme for the years 2001 to 2006.

including an increase in the pensions of mothers as it has been the case in Germany in 2014, this reform increases liabilities. This yields a higher rate of return while simultaneously the uncovered liabilities are increased as no additional subsidies are warranted and therefore assets do not change. Including uncovered liabilities makes it possible to estimate the part of the internal rate of return originating from transferring a financial burden to future contributors.

Second, we also include public subsidies from tax revenue estimated by the public contribution asset (as discussed in section 2.3.3.). Starting from the balancing equation (Equation 2), the cross-sectional internal rate of return is that very rate which ensures that the balancing equation holds.

Eq. 7 $$\frac{\boldsymbol{d(BF + CA + PCA + UL - PL)}}{\boldsymbol{dt}} = \mathbf{0}$$

Inserting Equations 5 and 6 and totally differentiating, we end up with Equation 8 describing the evolution of the balance sheet positions:

Eq. 8 $$\boldsymbol{TD * \left(\frac{dC}{dt} + \frac{dS}{dt}\right) + (C + S) * \frac{dTD}{dt} + \frac{dBF}{dt} + \frac{dUL}{dt} - \frac{dPL}{dt} = 0}$$

Pension liabilities increase due to the credited internal rate of return (*IRR*) on the existing liability and through paid contributions (*C*) and public subsidies (*S*) creating new liabilities. By contrast, it decreases with the amount of pensions paid out (*P*), as shown in Equation 9.

Eq. 9 $$\boldsymbol{\frac{dPL}{dt} = PL * IRR + C + S - P}$$

For a notional-defined contribution system (NDC) as in Sweden with paid contributions equaling hereby accrued liabilities this holds true.[29] For any other pension scheme with a non-actuarial pension formula, contributions may exceed the credited liabilities implying an implicit

[29] Actually this holds only true for Sweden from the base year accounting perspective if cross-sectional mortality is used to calculate liabilities. From a life-cycle perspective contributions may be smaller than hereby accrued liabilities due to increases in life expectancy. This fact will be shown later on.

tax or vice versa an implicit subsidy. If there exists such an implicit tax or subsidy equal to the difference between contributions and subsidies paid and credited liabilities, it will be included in the resulting IRR. This property is shown and discussed later on in more detail.

The buffer fund changes by the difference between contributions, subsidies and pension disbursement and the return on existing assets as shown in Equation 10.

Eq. 10 $$\frac{dBF}{dt} = BF * r + C + S - P$$

Inserting Equations 9 and 10 in Equation 8 and solving for the internal rate of return yields the following expression for the cross-sectional internal rate of return of the pension scheme.

Eq. 11 $$IRR = \underbrace{\frac{TD*\left(\frac{dC}{dt}+\frac{dS}{dt}\right)}{PL}}_{\substack{Changes\\ in\ contributions}} + \underbrace{\frac{(C+S)*\frac{dTD}{dt}}{PL}}_{\substack{Change\ in\\ turnover\ duration}} + \underbrace{\frac{BF*r}{PL}}_{\substack{Return\ on\\ financial\ assets}} + \underbrace{\frac{\frac{dUL}{dt}}{PL}}_{\substack{Changes\ in\\ uncovered\\ liabilities}}$$

As is obvious from Equation 11, the internal rate of return can be decomposed into four different components. The first term denotes changes in the financing of the pension scheme either due to contributions or public subsidies. The second term represents changes in the expected turnover duration and thus captures changes with respect to the population structure and income patterns. The third term denotes the funding component of the pension scheme, whereas the last component captures the change in the uncovered liabilities and thus also implicit taxes or subsidies on contributions, increases in longevity etc. It represents the part of the IRR not backed by assets and thus represents a financing burden of the pension scheme which will materialize sometime in the future.

It should be noted that pension indexation in the German pension scheme applies exactly in the same way to both pensions paid out and entitlements of the current working population. Therefore the IRR also refers exactly to both entitlements of pensioners (pensions paid) as well as to entitlements of non-pensioners.

Table 2: Components and estimation of the cross-sectional internal rate of return

	2006	2007	2008	2009	2010	2011	2012
$\frac{TD*\left(\frac{dC}{dt}+\frac{dS}{dt}\right)}{PL}$	4.28%	-1.86%	2.08%	0.88%	1.90%	1.54%	1.58%
$\frac{(C+S)*\frac{dTD}{dt}}{PL}$	-0.20%	0.89%	0.07%	-0.19%	0.30%	-0.01%	0.41%
$\frac{BF*r}{PL}$	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%
$\frac{\frac{dUL}{dt}}{PL}$	-2.44%	1.41%	-2.03%	0.85%	1.12%	0.81%	-0.75%
IRR **(nominal)**	1.65%	0.45%	0.12%	1.55%	3.32%	2.34%	1.24%
IRR **(real)**	-0.21%	-1.80%	-2.49%	1.33%	2.10%	-0.13%	-0.83%

Source: Own calculations based on official statistics from the German pension insurance (DRV). IRR (real) has been deflated using the Harmonized Index of Consumer Prices published by the Federal Statistical Office.

The nominal internal rate of return is positive throughout the observed period. However, in the years 2007 and 2008 it has been close to zero due to a decrease in contributions 2007 and a decrease of uncovered liabilities in 2008. These balance sheet positions are the main drivers of the IRR over the observed time horizon, but they are interdependent in two different ways. Pension indexation in Germany follows wage growth, but official data on wage growth is only available with a time lag. Thus pension levels and accordingly liabilities increase in line with lagged wage growth. As uncovered liabilities are the difference between assets and liabilities, each change in one of both positions simultaneously has an impact on the change in uncovered liabilities. While wage growth increases the contribution asset and simultaneously decreases uncovered liabilities in the same period, it leads to an increased pension indexation and thus increases uncovered liabilities in the next period. The funding component of the IRR resulting from financial returns is nearly neglectable due to the small size of the buffer fund.

It has to be noted that these estimates refer to the nominal internal rate of return. Additionally the real internal rate of return has been calculated by using the increase in the Harmonized Index of Consumer Prices as deflator. Except for the years 2009 and 2010 the real internal rate of return has been negative.

However, in general a pension scheme offers additional insurance with respect to different risks, which are not included in our estimates here. For example Gordon and Varian (1988) were the first to show that an unfunded pension scheme, representing a contract between generations, is able to provide risk-sharing with respect to income shocks between generations. With respect to different designs of pay-as-you-go pension schemes this has been further analyzed by Thøgersen (1998).[30] Some work has also been concentrating on the risk sharing of the automatic adjustment mechanisms in the German pension scheme. Knell (2010) analyzes the question how the internal rate of return is affected by demographic fluctuations with respect to different automatic adjustment mechanisms. He concludes that the German adjustment formula spreads demographic risk most equally among generations. Ludwig and Reiter (2010) find that the German adjustment formula spreads demographic risk more equally across cohorts than the second-best solution, explicitly modelling labor supply distortions by the contribution rate. These results underline the importance of risk sharing mechanisms of the German pension scheme, which can unfortunately not been taken into account by our estimates of the IRR.

Besides risk sharing properties across generations, every pension scheme paying annuities provides insurance against the risk of longevity by paying benefits until death even if life expectancy is higher than expected. On the one hand this feature exposes funded as well as

[30] Since then many authors have analyzed different dimensions of risk-sharing through an unfunded pension scheme, for example more recently Gottardi and Kubler (2011), Krueger and Kubler (2006), Andersen (2014) as well as Auerbach and Lee (2011), to name just a few of them.

unfunded pension schemes to a financing risk implied by (unexpected) increasing longevity. On the other hand increasing longevity positively contributes to the internal rate of return of the pension scheme by increasing expected benefits. Knell (2016) analyzes the impact of increasing longevity on IRR from a life-cycle perspective, considering two possible policy reactions to balance the pension scheme, either increasing the retirement age or cutting pension levels.[31]

Likewise, we are interested in the impact of decreasing mortality on the cross-sectional IRR. From the cross-sectional perspective taken throughout this article, we try to assess the part of the IRR resulting from changes in cross-sectional mortality by means of comparative statics. Starting with Equation 11, the change in liabilities can be decomposed into changes in mortality $\boldsymbol{\gamma}$ and all other changes, consisting of changing demographic structure, pension indexation, differences between contributions and pensions paid et cetera, denoted $\boldsymbol{\mu}$. Likewise, the IRR can be subdivided into the change in mortality and other changes as shown in Equation 12.[32]

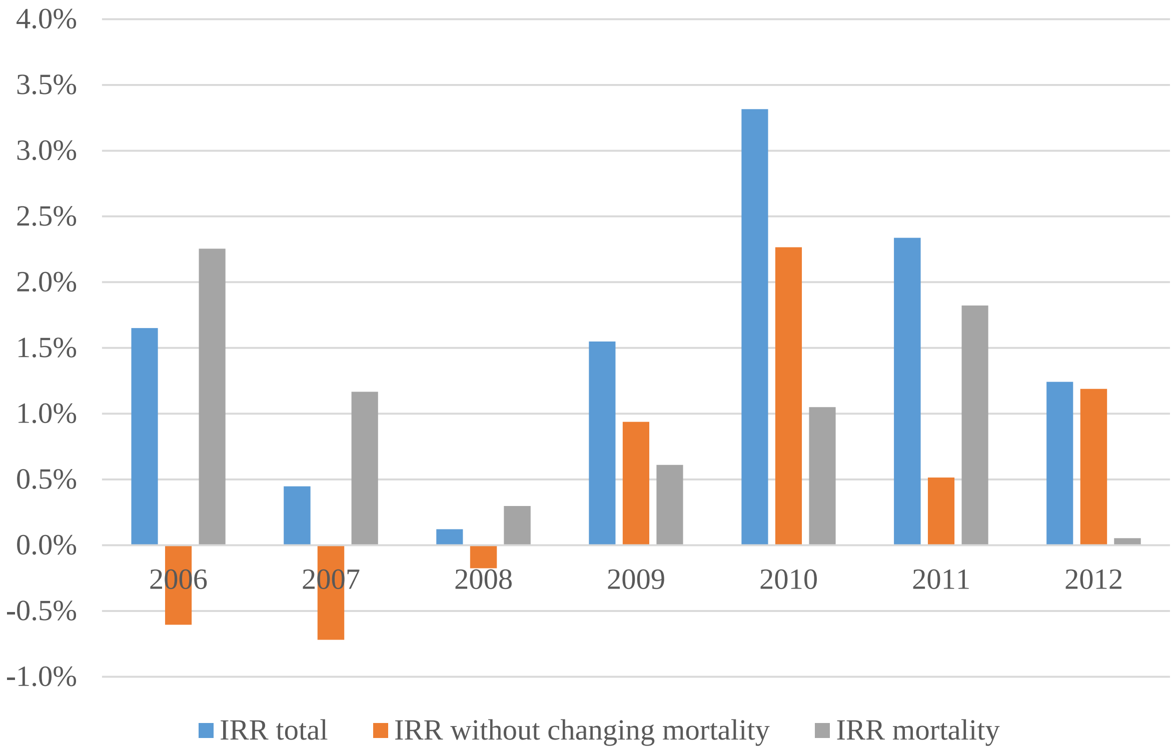
Eq. 12 $$\underbrace{\frac{dPL}{d\mu} * \frac{d\mu}{dt}}_{\substack{other \\ changes}} + \underbrace{\frac{dPL}{d\gamma} * \frac{d\gamma}{dt}}_{\substack{changing \\ mortality}} = \boldsymbol{PL} * (\boldsymbol{IRR_{\mu}} + \boldsymbol{IRR_{\gamma}}) + \boldsymbol{C} + \boldsymbol{S} - \boldsymbol{P}$$

With constant mortality ($\frac{d\gamma}{dt} = 0$) the internal rate of return resulting from changing mortality $\boldsymbol{IRR_{\gamma}}$ is by definition equal to zero. Thus it is possible to estimate $\boldsymbol{IRR_{\mu}}$ by calculating the change in liabilities holding mortality rates constant between two respective years. The IRR resulting from changing mortality can finally be estimated as residual between overall IRR and IRR resulting from all other changes.

[31] His results suggest, that with a simultaneously increasing retirement age the positive effect of increasing longevity on the IRR is nearly twice as large as if pension levels are cut.

[32] Changes in pension liabilities due to non-actuarial fairness of contributions are captured by other changes and thus are included within IRR_{μ}.

Figure 1: Mortality driven internal rate of return



Source: Own estimations.

As is obvious from Figure 1, changes in mortality contribute significantly to the internal rate of return of the pension scheme. While the IRR arising from all other changes was even negative in the years 2006 to 2008, decreases in mortality turned the overall IRR positive. In 2006 and 2011 decreasing mortality increased the total IRR by about two percentage points. Thus ignoring changes in mortality in estimating internal rates of return of a pension scheme may lead to a significant downward bias of the results. These results can be interpreted as the insurance value against longevity of the pension insurance scheme.

The previously presented aggregate internal rate of return builds on the change of aggregate pension liabilities. However, this aggregate internal rate of return consists of two non-separable parts, the cross-sectional internal rate of return on pension liabilities (past contributions) and the cross-sectional internal rate on current contributions to the pension scheme. This issue is illustrated in the following section using a simple overlapping generations model with changing life expectancy.

We simultaneously consider three generations alive, with the two youngest generations working full-time, earning wage w and being subject to a contribution rate $\boldsymbol{\theta}$ of the pension scheme. Then the cross sectional internal rates of return are determined as follows in order to link contributions to the resulting pension payments:

Eq. 13 $$\underbrace{\boldsymbol{PL_t}}_{\substack{Pension\\liability}} = \underbrace{\boldsymbol{p_t}}_{\substack{pension\\benefit}} * \underbrace{\boldsymbol{\gamma_t}}_{\substack{life\\expectancy}}$$

$$= \boldsymbol{\theta_{t-2} w_{t-2} (1 + i_{t-1})(1 + i_t) + \theta_{t-1} w_{t-1} (1 + i_t)}$$

The aggregate pension entitlement or liability in period t consists of the sum of accrued entitlements when young (Y) and middle-aged (M), changing with pension/entitlement indexation $(1 + d_t)$ and changing life expectancy $(1 + \varphi_t)$. We consider only actual pension/entitlement indexation, as in the short run (annual perspective) these may differ from their long run properties in simple models (contribution growth) due to intertemporal shifting of unfunded liabilities.

Eq. 14 $$\underbrace{\boldsymbol{PL_t}}_{\substack{Pension\\liability}} = \boldsymbol{p^Y_{t-2} * \gamma_{t-2} * (1 + d_{t-1})(1 + d_t)(1 + \varphi_{t-1})(1 + \varphi_t) + p^M_{t-1}}$$

$$\boldsymbol{* \gamma_{t-1}(1 + d_t)(1 + \varphi_t)}$$

The internal rate of return for the first period can then be calculated straightforward as the respective rate, which balances previous contributions and current pension entitlements as shown in Equation 15:

Eq. 15 $$\boldsymbol{\theta_{t-2} w_{t-2}(1 + i_{t-1}) = p^Y_{t-1} * \gamma_{t-1}}$$

Writing the pension liability as the pension liability of the previous period and considering pension indexation and changes in mortality, Equation 15 can be rewritten as Equation 16.

Eq. 16

$$(1+i_{t-1}) = \underbrace{\frac{p^{Y}_{t-2} * \gamma_{t-2}}{\theta_{t-2} w_{t-2}}}_{accrual\ rate} * \underbrace{(1+d_{t-1})(1+\varphi_{t-1})}_{change\ of\ entitlements}$$

It consists of the general change of the accrued pension entitlement due to entitlement/pension indexation and changes in life expectancy multiplied by the accrual rate of the respective period, namely the ratio of the accrued pension entitlement to contributions paid. This ratio could also be interpreted as showing the actuarial fairness of contributions in that period. From a cross-sectional perspective the internal rate of return in the second period refers to both pension entitlements accrued before and contributions in the second period. Inserting Equations 14 and 16 in Equation 13 finally yields the following expression for the internal rate of return in the second period presented by Equation 17.

Eq. 17

$$(1+i_{t}) = (1+d_{t})(1+\varphi_{t})[1+\frac{p^{M}_{t-1} * \gamma_{t-1} - \theta_{t-1} w_{t-1}}{\theta_{t-2} w_{t-2}(1+i_{t-1}) + \theta_{t-1} w_{t-1}}]$$

If in the second period no additional pension entitlements are accrued and no contributions are paid, then the internal rate of return on the already existing liability just equals the product of pension indexation and changes in life expectancy. If additional entitlements are accrued, the internal rate of return on previous entitlements and current contributions consists of the before mentioned product of pension indexation and life expectancy changes times one plus the actuarial difference between contributions and accrued entitlements in the second period divided by the total assessment basis for the internal rate of return, current entitlements and current contributions.

Thus, from a cross-sectional perspective internal rates of return always refer to changes in pension entitlements accrued before that period and additionally capture any actuarial differences between contributions paid and entitlements accrued during that period. If accrued pension entitlements exceed contributions, this will also alter the internal rate of return during this period and vice versa. If institutional features of the pension scheme as e.g. the regular retirement age, the pension formula etc. do not change, increasing longevity does not only lead to a higher growth in existing pension entitlements but also increases the pension entitlement accrual rate, given that the pension amount is defined as a percentage of the wage level and not subject to changes due to increasing longevity.

The analytics of this simple model may easily be transferred to our estimation of the aggregate internal rate of return. The cross-sectional aggregate internal rate of return of the pension scheme captures exactly the above shown features. However, from this macro perspective total contributions to the pension scheme as well as aggregate pension entitlements to both contributors and pensioners are taken into account. The largest fraction of the internal rate of return results from changes in pension or entitlements indexation as well as from changes in mortality and thus life expectancy. Additionally the non-actuarial part of contributions during the respective year plays a minor role, because current contributions can almost be neglected compared to total pension entitlements.[33] Thus, our estimation of the cross-sectional internal rate of return captures changes in existing aggregate pension entitlements and changes in pension entitlements due to total contributions during the specific year.

[33] Contributions including public subsidies amount only to a fraction of about 1/30 of aggregate pension entitlements.

3.4. Implicit Tax

In the following section we will turn our attention to the implicit tax of the German statutory pension scheme over the observed period. Starting with Lüdecke (1988), there have been many articles focusing on the implicit tax levied on contributions to pay-as-you-go pension schemes. Implicit taxes on contributions arise from the fact that individuals are forced to save in the pension scheme with the internal rate of return being smaller than a comparable market rate of interest. Sinn (2000) argues that also without efficiency differences an implicit tax may exist in order to refinance the implicit debt arising from introductory gains of the pension scheme. Most studies (e.g. Sinn (2000), Fenge and Werding (2004) and Schnabel (1998)) concentrate on the implicit tax share of life-cycle contributions, while from our cross-sectional analysis we are interested in annual tax rates , as analyzed by Beckmann (2000) or Fenge et al. (2002). Our model is close to that used by Fenge et al. (2016), differing in two dimensions. First, we refrain from solving for the overall budget constraint of the pension scheme, as from an annual perspective accrued pension entitlements must not be backed up by contributions but may be shifted to the future. Second, we explicitly differentiate between the pension or pension entitlement indexation and the increase in total entitlements due to increasing longevity. While Beckmann (2000) and Fenge et al. (2002) relate the implicit annual tax rates to contributions, we additionally show that from a cross-sectional accounting perspective implicit annual tax rates may also be considered as levied on past contributions representing current pension entitlements. In order to do so, we make use of the previously introduced three-period overlapping generations model. From a cohort's perspective we may first consider the absolute implicit tax from the first period to the second, defined as the difference between contributions and discounted pension entitlement in the second period (Equation 18).

Eq. 18 $$\boldsymbol{T_{t-1} = \theta_{t-2} w_{t-2} - \frac{p_{t-1}^{Y} * \gamma_{t-1}}{(1 + r_{t-1})}}$$

Relating the lump sum tax to the tax base, here dividing by past contributions, the implicit tax rate of the first period is determined by Equation 19. Thus, the implicit tax rate on first period contributions equals one minus the period specific accrual rate times the fraction of pension entitlement indexation to the rate of return on risk-free financial assets $\boldsymbol{r_t}$.

Eq. 19

$$\boldsymbol{\tau_{t-1} = 1 - \frac{p^Y_{t-2} * \gamma_{t-1}}{\theta_{t-2} w_{t-2}} * \frac{(1 + d_{t-1})(1 + \varphi_{t-1})}{(1 + r_{t-1})}}$$

The implicit tax in the second period equals the entitlements at the beginning of the respective period (already accrued entitlements) and the contributions of that period minus the present value of the full pension entitlement to be paid out in the last period. Rebasing the pension entitlements to t-1 and rearranging, the implicit lump sum tax of the second period can be written as Equation 20.

Eq. 20

$$\boldsymbol{T_t = p^Y_{t-1} * \gamma_{t-1} + \theta_{t-1} w_{t-1} - \frac{p^Y_t * \gamma_t}{(1 + r_t)} - - \frac{p^M_t * \gamma_t}{(1 + r_t)}}$$

$$\boldsymbol{T_t = p^Y_{t-1} * \gamma_{t-1}(1 - \frac{(1 + d_t)(1 + \varphi_t)}{(1 + r_t)}) + \theta_{t-1} w_{t-1}(1 - \frac{p^M_{t-1} * \gamma_{t-1}}{\theta_{t-1} w_{t-1}} * \frac{(1 + d_t)(1 + \varphi_t)}{(1 + r_t)})}$$

Dividing the implicit tax by the tax base, namely existing pension wealth and contributions in t-1, yields the corresponding implicit tax rate in that period. From Equation 21 it becomes clear that from a cross-sectional perspective the annual implicit tax rate of a pension scheme is determined by two different components. First, an “implicit wealth tax” is levied on already accrued pension entitlements/liabilities arising from the difference between the change of the pension entitlement (indexation times the increase of life expectancy) and the market rate of return. Second, contributions during that year are subject to an implicit tax resulting from the before mentioned differences in pension entitlement indexation and the market rate of return times the accrual rate. The average implicit tax rate of the pension scheme finally equals one minus these two tax components divided by the tax base, already existing entitlements and current contributions. In line with Beckmann (2000) and Uebelmesser (2004), these results imply a decreasing implicit tax share on contributions over the life-cycle if we assume that the market rate of return is larger than indexation of pension entitlements throughout the life cycle. Early contributions and the hereby accrued pension entitlement are thus subject to the "implicit wealth tax" each year, resulting in a higher tax share. By contrast, the implicit tax rate on contributions late in life may become negative even if the market rate of return exceeds the entitlement indexation due to a non-actuarial fair accrual rate larger than one.

Eq. 21
$$\boldsymbol{\tau_t = \frac{T_t}{p^y_{t-1}\gamma_{t-1} + \theta_{t-1}w_{t-1}}}$$
$$\boldsymbol{= 1 - \frac{\overbrace{p^Y_{t-1} * \gamma_{t-1} * \frac{(1+d_t)(1+\varphi_t)}{(1+r_t)}}^{Tax\ on\ existing\ pension\ liability/wealth} + \overbrace{\theta_{t-1}w_{t-1} * \frac{p^M_{t-1} * \gamma_{t-1}}{\theta_{t-1}w_{t-1}} \frac{(1+d_t)(1+\varphi_t)}{(1+r_t)}}^{Tax\ on\ contributions\ during\ year}}{p^Y_{t-1} * \gamma_{t-1} + \theta_{t-1}w_{t-1}}}.$$

Substituting Equation 21 by Equation 17 and rearranging, one can show that the implicit annual tax rate, equals the well-known results by Sinn (2000) shown in Equation 22, except for the fact that from our accounting perspective the internal rate of return as well as the implicit tax rate refer not only to contributions but to existing pension liabilities and contributions.[34]

Eq. 22
$$\boldsymbol{\tau_t = 1 - \frac{1+i_t}{1+r_t}}$$

Hence, with existing pension wealth exceeding contributions to the pension scheme by a factor of about 30, this implicit tax rate describes more exactly the implicit wealth tax part than the implicit tax rate on contributions during the year. What becomes obvious from our result is the fact that timing of these implicit wealth taxes matters. This results from the fact that total

[34] We can only estimate an average tax rate for both components, as we are not able to distinguish between indexation and actuarial fairness properties of the internal rate of return for current contributions.

pension wealth is lowest at the start of the contributions career and highest before retirement. Hence, high implicit tax rates shortly before retirement imply a higher aggregate implicit tax rate than if tax rates are higher at the beginning of the working career.

Using our estimates of the cross-sectional internal rate of return, we first need to clarify what is the appropriate market rate of return in order to estimate the cross-sectional implicit tax levied on pension entitlements and contributions. As the market rate of return is crucial for estimating the empirical implicit tax rate of the pension scheme, this choice will influence the results significantly. However, every choice for the benchmark market rate of return has some shortcomings and thus our empirical estimates of the implicit tax rate might rather be interpreted as best guess.

A risk-free asset as for example the return on government bonds would represent a common choice. Hence, the appropriate maturity of these government bonds should in principle equal the average maturity of any pension entitlement. This is measured by the turnover duration and should approximately equal thirty years. Accordingly, we use the yield on outstanding German government bonds with a maturity of fifteen to thirty years as published by the Bundesbank. Additionally, we also use a more common maturity of ten years for sensitivity reasons. Generally, one has to keep in mind that inherent intergenerational risk-sharing properties of the pension scheme can unfortunately not be considered in our framework leading to an overestimation of the implicit tax rate of the pension scheme.

Table 3: Cross-sectional implicit tax rates

	2006	2007	2008	2009	2010	2011	2012
Yield on government bonds 10-year maturity	3.35%	3.76%	4.22%	3.98%	3.22%	2.74%	2.61%
Implicit tax rate	2.03%	3.62%	3.71%	1.62%	-0.56%	0.27%	0.25%
Yield on government bonds 15-30 years maturity	4.01%	4.45%	4.52%	4.13%	3.43%	3.28%	2.25%
Implicit tax rate	2.27%	3.83%	4.21%	2.48%	0.11%	0.91%	0.99%

Source: Own calculations based on official statistics from the German pension insurance (DRV) and financial statistics published by the Bundesbank.

The empirical estimates for the implicit tax rate are presented in Table 3 using two different maturities of outstanding government bonds: ten years and fifteen to thirty years. Not surprisingly, longer maturities yield higher rates of return and thus larger implicit tax rates. Although our period of observation is only covering seven years, implicit cross-sectional tax rates show a huge variation ranging from 4.2 to 0.11 percent using the longer maturity. This large variation results from two facts. First, yields on government bonds are significantly decreasing over the observed time period. Second, the large fraction of this variation is coming from the volatile cross-sectional rate of return of the pension scheme (see Table 2). Applying the yield on outstanding government bonds with a maturity of ten years even leads to a negative overall implicit tax rate in the year 2010 of minus 0.56 percent. In the light of currently low interest rates, negative implicit tax rates are not unlikely to be also observed over the next years.

4. Implications, limitations and outlook

In this article we extend and apply the framework of Settergren and Mikula (2005) for accounting of paygo pension schemes to the German statutory pension insurance. We estimate a balance sheet for the years 2005 to 2012 considering old-age, disability and survivors' pension entitlements. Our results suggest that the German pension insurance is nearly balanced in a "Swedish" sense with a funding ratio of about 90 percent. This funding ratio stays pretty constant over the whole period, giving room to the assumption that, although different, the German sustainability factor also leads to a balanced pension scheme in the Swedish sense. While the final balancing effect of the sustainability factor has to be left to further research, we use the change in balancing items over time in order to estimate the average cross-sectional internal rate of return of the pension scheme itself. The results reveal that the cross-sectional internal rate of return is pretty volatile, ranging from close to zero to over three percent. Additionally it becomes obvious that from a financing perspective, the largest parts of the internal rate of return are either "financed" by changing contributions or a change in uncovered liabilities. From an expenditure perspective, we are the first to ask furthermore how large is the share of the IRR resulting from increasing longevity. The results reveal that also mortality changes are pretty volatile over the observed period, but if they take place, they account for a major part of the internal rate of return.

Against the backdrop of distortions induced by the pension scheme, we develop a simple model in order to analyze the implicit tax of a pension scheme from a cross-sectional perspective. Our main results are that the largest part of the implicit tax can be interpreted as an "implicit wealth tax" on existing pension wealth and the smaller part as an implicit tax on current contributions. The implicit wealth tax on earlier accrued pension wealth depends on differences in indexation of pension wealth and financial assets. The implicit tax share of contributions consists of the former part and additionally on the actuarial fairness of contributions with respect to the hereby accrued pension wealth. Thus, implicit tax rates on contributions may become positive later in life even if the market rate of return exceeds indexation of pension wealth due to missing actuarial fairness. Furthermore, if the market rate of return exceeds indexation of pension wealth, we observe falling implicit tax shares of contributions to the pension scheme over the life-cycle due to multiple taxation of existing pension wealth. A second implication of these results is the fact that timing of implicit taxes matters, as accrued pension wealth increases with contribution years.

Using yields on outstanding German government bonds with a maturity of 15 to 30 years, we estimate annual implicit tax rates of between 0.11 and 4.5 percent. In the light of currently low interest rates, the implicit tax rates may even become negative for the following years. However, one has to keep in mind that pay-as-you-go financed pension schemes may include various risk-sharing properties, which we are unfortunately not able to take into account in our presented framework. Hence, this issue must be left for further research.

References

Aaron, H. (1966), The Social Insurance Paradox, in: *The Canadian Journal of Economics and Political Science*, 32(3), 371-374.

Andersen, T. (2014), Intergenerational redistribution and risk sharing with changing longevity, *Journal of Economics*, 111(1), 1-27.

Auerbach, A., J. Gokhale und L. Kotlikoff (1991), »Generational Accounts: a meaningful alternative to deficit accounting«, *Tax policy and the economy* 5, 55–110.

Auerbach, A., J. Gokhale und L. Kotlikoff (1994), »Generational Accounts: a meaningful way to evaluate fiscal policy«, *Journal of Economic Perspectives*8, 73–94.

Auerbach, A. and R. Lee (2011), Welfare and generational equity in sustainable unfunded pension schemes, *Journal of Public Economics*, 95(1-2), 16-27.

Beckmann, K. (2000), A Note on the Tax Rate Implicit in Contributions to Pay-As-You-Go Public Pension Systems, *Finanzarchiv / Public Finance Analysis*, 57(1), 63-76.

Billig, A. and J.-C. Menard (2013), Actuarial balance sheet as a tool to assess the sustainability of social security pension schemes, in: *International Social Security Review*, 66(2), 31-52.

Board of Trustees, Federal old-Age and Survivors Insurance and Disability Insurance Trust Funds (BOT) (2016), 2016 Annual Report, Government Printing Office, Washington D.C.

Bundesamt für Sozialversicherungen (BSV) (2015), AHV-Statistik 2014 – Tabellenteil, Bern.

Bundesrat (2013), Gesamtsicht über die Finanzierungsperspektiven der Sozialversicherungen bis 2035, Bern.

Boado-Penas, M., S. Valdes-Prieto and C. Vidal-Melia (2008), The Actuarial Balance Sheet for Pay-As-You-Go Finance: Solvency Indicators for Spain and Sweden, in: *Fiscal Studies*, 29(1), 89-134.Deutsche Rentenversicherung (2015), Jahresbericht 2014, Berlin.

Eurostat (2011), Technical compilation guide on pensions data in national accounts, *Eurostat Methodologies & Working papers,* Luxembourg.

Fenge, R., S. Uebelmesser and M. Werding (2002), Second-best properties of implicit social security taxes: Theory and evidence, *Cesifo Working Paper No.* 743, Munich.

Fenge, R. and M. Werding (2003), Ageing and fiscal imbalances across generations: concepts of measurement, *Cesifo Working Paper No.* 842, Munich.

Fenge, R. and M. Werding (2004), Ageing and the Tax Implied in Public Pension Schemes: Simulations for Selected OECD Countries, *Fiscal Studies*, 25(2), 159-200.

Geyer, J. and V. Steiner (2014), Future public pensions and changing employment patterns across birth cohorts, *Journal of Pension Economics and Finance*, 13(2), 172-209.

Gordon, R.H. and H.R. Varian (1988), Intergenerational risk sharing, *Journal of Public Economics*, 37(2), 185-202.

Gottardi, P. and F. Kubler (2011), Social Security and risk sharing, *Journal of Economic Theory*, 146(3), 1078-1106.

Government of Canada (2015), Annual Report of the Canada Pension Plan 2014-2015,

Holzmann, R., R. Palacios and A. Zviniene (2004), Implicit Pension Debt: Issues, Measurement and Scope in International Perspective, *Social Protection Discussion Paper* No. 0403, The World Bank.

Kaier, K. and C. Müller (2015), New figures on unfunded public pensions entitlements across Europe: concept, results and applications, in: *Empirica*, 42(4), 865-895.

Knell, M. (2010), How automatic adjustment factors affect the internal rate of return of PAYG pension systems, *Journal of Pension Economics and Finance*, 9(1), 1-23.

Knell, M. (2016), The return on social security with increasing longevity, *Macroeconomic Dynamics*, FirstView, DOI: http://dx.doi.org/10.1017/S1365100515000589.

Krueger, D. and F. Kubler (2006), Pareto-Improving Social Security Reform when Financial Markets are incomplete!? , *American Economic Review*, 96(3), 737-755.

Lee, R. D. (1994), Population Age Structure, Intergenerational Transfer, and Wealth: A New Approach, With Applications to the United States, *The Journal of Human Resources*, 29(4), 1027-1063.

Ludwig, A. and M. Reiter (2010), Sharing Demographic Risk – Who is afraid of the Baby Bust?, American Economic Journal: Economic Policy, 2(4), 83-118.

Lüdeke, R. (1988), Staatsverschuldung, intergenerative Redistribution und umlagefinanzierte gesetzliche Rentenversicherung: Eine andere Sicht der Lasten durch ein negatives Bevölkerungswachstum, in: J. Klaus und P. Klemmer (eds.), Wirtschaftliche Strukturprobleme und Soziale Fragen , Duncker und Humblot, Berlin.

Metzger, C. (2016), Accounting of pay-as-you-go pension schemes using accrued-to-date liabilities – An example for Switzerland, *FZG Discussion Paper* No. 59, Research Center for Generational Contracts (FZG).

OECD (2013), Pensions at a Glance 2013: OECD and G20 indicators, *OECD Publishing*, Paris.

Schnabel, R. (1998), Rates of Return of the German Pay-As-You-Go Pension System, *Finanzarchiv / Public Finance Analysis*, 55(3), 374-399.

Settergren, O. and B. Mikula (2005), The rate of return of pay-as-you-go pension systems: a more exact consumption-loan model of interest, in: *Journal of Pension Economics and Finance*, 4(2), 115-138.

Sinn, H.-W. (2000), Why a funded pension is useful and why it is not, in: *International Tax and Public Finance*, 7(4), 389-410.

Swedish Pension Agency (2014), Orange Report - Annual Report of the Swedish Pension System 2013, Stockholm.

Thøgersen, Ø. (1998), A note on intergenerational risk-sharing and the design of pay-as-you-go pension programs, *Journal of Population Economics*, 11(3), 373-378.

Uebelmesser, S. (2004), *Unfunded Pension Systems: Ageing and Migration*, Contributions to Economic Analysis 264, North-Holland: Amsterdam

Vidal-Melia, C. and M. Boado-Penas (2013), Compiling the actuarial balance for pay-as-you-go pension systems. Is it better to use the hidden asset or the contribution asset, in: *Applied Economics*, 45(10), 1303-1320.

Forschungszentrum Generationenverträge
Albert-Ludwigs-Universität Freiburg
Bertoldstraße 17
79098 Freiburg
Fon 0761 . 203 23 54
Fax 0761 . 203 22 90
www.generationenvertraege.de
info@generationenvertraege.de