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Assessing Social Security: Some Useful Results

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Assessing Social Security: Some Useful Results

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Abstract

This paper presents a dense summary of the most fundamental results on the issue of PAYG Social Security. They are presented in a simple and insightful analytical framework by working from the budget constraint of the household. The results presented in detail are a representation of the Aaron-Theorem, the equivalence of debt and Social Security, and a proof that under certain conditions the burden of Social Security on the current young and all future generations equals the benefit payments to the current old. Finally, I include uncertainty in the analysis and touch on the intergenerational risk-sharing implications of Social Security.

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I Introduction

The future design of Social Security is at the heart of the political debate in almost all developed countries. This debate has been lead and accompanied by a tremendous amount of research on the economics of "Privatizing Social Security". A large part of this research has focused on country specific problems and on current reform proposals in the different countries.

In contrast to this important literature I try to consolidate the fundamental results one should know when following and participating in this debate. This is done in an analytic yet easy to follow framework. In most cases the results can be shown just by comparing the life-cycle-budget constraints of the individual household under the different policies. For some results it is necessary to consider the general equilibrium effects, but this is done in a quite general framework with only the very basic assumptions on household behavior, market efficiency, and production technology.

All in all this paper assesses the basic results from the Aaron-Theorem to intergenerational risk sharing in one comprehensive framework focusing on the economic basics without going into institutional details of specific countries. At the same time the underlying assumptions and their implications are always discussed. The single arguments are thereby put into a broader perspective.

The analytical framework is a two period overlapping generations economy in three settings: an economy without government activity, an economy with pay-as-you-go (PAYG) Social Security and an economy with public debt. In Section 2 the government absent economy is compared with the Social Security case. In Section 3, I show the parallels of Social Security and public debt. In Section 4 a straight forward proof that the present value of the perpetual implicit tax of Social Security equals the current benefit payments is presented. However this is only true for the case of a small open economy and in the dynamic efficient region. The importance and relevance of these assumptions – especially the dynamic efficiency assumption – are discussed and I point to the possible welfare increasing role asset bubbles can have in the dynamic inefficient region in place of Social Security or public debt. The problem of switching regimes is also touched upon. In Section 5, I focus on the intergenerational risk sharing component Social Security can offer in an economy with uncertainty that is not available in the pure market solution of the economy.

II PAYG versus funded Social Security under certainty

II.1 The framework

The standard framework to investigate questions concerning Social Security is a model of overlapping generations (OLG) as put forward by Samuelson (1958) and Diamond (1965). In order to reduce the complexity I consider an economy in which one generation consisting of N_t^y individuals who are working in period t and are retired in period $t+1$. So in every period there are two cohorts alive. The young who represents the current working population and the old who represents the current retired part of the population. The growth rate of the population is denoted n_t and so the demographic process is described by:

$$N_t^y = N_{t-1}^y(1+n_{t-1}). \tag{1}$$

II.2 The budget constraint under a funded system

The agent works a fixed number of hours, which I scale to unity for simplicity, and earns wage income w_t^y. This labor income is divided between consumption when young c_t^y and savings s_t^y for retirement. These savings will then be invested in capital.

$$c_t^y = w_t^y - s_t^y \tag{2}$$

When old this agent will want to consume the return of his savings (initial investment plus interest minus depreciation). Since by assumption the individual does not work when old, the return of his savings will equal the entire consumption when old.

$$c_{t+1}^o = (1+r_{t+1})s_t^y \tag{3}$$

Solving the second period budget constraint for s_t^y and substituting into the first period budget constraint produces the life-cycle budget constraint of the representative household:

$$c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} = w_t^y. \tag{4}$$

This represents the very simplest setup of an overlapping generations economy without government activity. In a next step I will introduce a tax-transfer system that taxes the young and gives a transfer to the old. Such a scheme corresponds to the basic setup of pay-as-you-go organized Social Security. I will then distinguish the two schemes by comparing the respective life-cycle budget constraint of the representative households under the alternative schemes.

II.3 The budget constraint under PAYG Social Security

I now introduce a mandatory PAYG system. The government, in form of a Social Security authority, collects lump-sum taxes τ^y from the working population and distributes transfers tr^o to the old individuals. In a matured[1] system the period budget constraints equivalent to equations 2 and 3 are then:[2]

$$c^{y,ss} = w_t^y - s_t^{y,ss} - \tau_t^y, \text{ and} \tag{5}$$

$$c_{t+1}^{o,ss} = (1 + r_{t+1})s_t^{y,ss} + tr_{t+1}^o. \tag{6}$$

By combining the two periods budget constraints as before we get the life-cycle budget constraint of the household in an economy with mandatory Social Security:

$$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = w_t^y - \tau_t^y + \frac{tr_{t+1}^o}{1 + r_{t+1}}. \tag{7}$$

For simplicity let us consider a Social Security policy where the authority is not allowed to run any debt and has to guarantee a fixed contribution rate τ for all times. Under such a policy all transfers to the old tr_t^o are always covered by the tax payments of the current young. The budget constraint of the Social Security is therefore $tr_t^o N_t^o = \tau^y N_t^y$. Since the old generation in t is equal to the young generation $t-1$ and so $\frac{N_t^y}{N_t^o} = \frac{N_t^y}{N_{t-1}^y} = 1 + n_{t-1}$ we can write the Social Security budget constraint in per capita terms:

$$tr_t^o = \tau(1 + n_{t-1}). \tag{8}$$

Using the Social Security budget constraint given in equation 8, the life-cycle budget constraint of the individuals (equation 7) can be rewritten as:

$$c_t^y + \frac{c_{t+1}^o}{1 + r_{t+1}} = w_t^y - \tau^y \cdot \frac{r_{t+1} - n_t}{1 + r_{t+1}}. \tag{9}$$

This is a version of Aaron's well-known equation (see Aaron (1966)) that states that the yield of a PAYG Social Security system equals the growth rate of the population[3]

[1] A system is matured when the impact of the Social Security scheme on the life-cycle budget constraint is identical for all living generations. In this case the system is matured after one period, when the old generation receiving benefits has already paid contributions. In reality even minor changes of Social Security can often take over 60 years to mature.

[2] For an easy distinction between the various schemes I adopt the superscript ss for Social Security and will later use superscript pd for variables concerning the public debt scheme.

[3] I assume a zero productivity growth rate here. Therefore the yield of PAYG Social Security is only the growth rate of the population n. Depending of the implementation of the Social Security scheme the benefits of technological progress can or cannot be shared with the old population. In the simplest case

while the return of a funded system equals the interest rate which in turn should equal the marginal product of capital. This is exactly what can be seen from comparing equations 9 and 4: The effect on the individual life-cycle resources of a forced participation in the Social Security scheme equals $-\tau_t^y \frac{(r_{t+1}-n_t)}{1+r_{t+1}}$ since τ is the amount of resources that have to be contributed to the Social Security system and $r-n$ is the difference in yields of the funded system and the obligatory PAYG system. For now I will assume that $r > n$ and so the PAYG system puts a burden on the households by reducing the life-cycle resources (and therefore it is obvious that a Social Security scheme can only be implemented as a compulsory system). The difference in yields has often been called the implicit tax component of PAYG Social Security schemes. The economic requirements under which the assumption $r > n$ holds and the implications of this will be briefly discussed in section IV.2. In Propositions 1, I summarize the results so far:

Proposition 1 *The difference in yields of a PAYG Social Security system in comparison to a funded system equals $n_t - r_{t+1}$. For $r > n$ and a compulsory lump-sum contribution τ to the PAYG Social Security at date t the loss of life-cycle resources to the representative household equals $\tau\frac{r_{t+1}-n_t}{1+r_{t+1}}$ in present value terms.*

III The equivalence of public debt and Social Security

There has been a number of authors that have shown that public debt is equivalent to a Social Security scheme.[4] Here I show this argument first in the insightful way of looking solely at the budget constraint on the individual level. In present value terms the life-cycle budget constraint of the household will be identical given a specific policy rule.

In order to proof that this equivalence will also hold in a macroeconomic context, one has to show that even though the saving behavior of the household differs in the two respective schemes, macroeconomic saving will nevertheless be equivalent under both systems. This is due to the government absorbing part of the household savings in order to issue the public debt. On a macroeconomic level the net savings rate of households and government is then equal to the households savings in the Social Security case.

the old generation benefits completely from technological progress and in the setting of the neoclassical growth model one could define $1+\hat{n} \equiv (1+n)(1+x) \approx 1+n+x$ where x is the technological progress. The yield of the PAYG system would then be the total growth rate of the economy $\hat{n}$.

[4] Among them Raffelhüschen (1989), Gale (1990) and Bohn (1997). Discussing the U.S. reform proposals of Social Security of the mid 1990s Bohn (1997) states a number of results on neutrality of taxes and Social Security contributions, and of debt financed trust funds and Social Security.

III.1 An economy with public debt

Consider an economy with existing public debt B_t but no current public expenditures. Then the public deficit D_t equals the difference between interest payments on existing debt $r_t B_t$ and tax revenue T_t. Since next period public debt is the sum of existing debt and public deficit we can write the public sector budget constraint as $D_t = B_{t+1} - B_t = r_t B_t - T_t$. Consider the case where the government levies a lump-sum tax ϕ_t^o on the old generation so that tax revenue equals $T_t = \phi_t^o N_t^o$. If the government follows a policy rule under which the debt per old capita $b_t \equiv \frac{B_t}{N_t^o}$ is kept constant ($b_t = b_{t+i} = b \quad \forall i$) the public sector budget constraint can be solved for the value of the lump-sum tax:[5]

$$\phi_t^o = (r_t - n_{t-1})b. \tag{10}$$

III.2 Equivalence in a microeconomic context

For comparing Social Security and public debt in a microeconomic context, one has to distinguish between the effect on the life-cycle resources and consumption on the one side and the reaction of savings on the other side. In this section, I will show first that for a specific size of public debt, the effects of public debt and of Social Security on the life-cycle resources of the households are equivalent. In the following section, I will then prove that the macroeconomic assumptions on capital stock, wage, and interest rates necessary for the microeconomic equivalence are in fact true.

Assuming for now that wages and interest rates are the same under both systems it is straightforward to say that consumption when young and old will also be identical under both regimes.[6] Even though life-cycle resources and the consumption paths are equivalent, private savings will differ.

Again I start out from the budget constraint of the households. Since no taxes have to be paid in the first period the entire labor income is distributed between consumption and savings.

$$c_t^{y,pd} = w_t^y - s_t^{y,pd} \tag{11}$$

The gross return on savings will be used to finance consumption and the lump-sum tax

[5]Note that $N_{t+1}^o / N_t^o = N_t^y / N_{t-1}^y = (1 + n_{t-1})$ follows from the underlying demographic process.

[6]The timing of tax- and contribution-payments is not identical as taxes are levied in the second period but contributions in the first period. In a more realistic setting one would also have to assume that the households are not faced by credit constraints so that they can freely shift life-cycle resources between periods. In our example this assumption is only necessary for the case where second period consumption is smaller than the transfers of the Social Security system.

payable when old. So the second period budget constraint equals equation 12.

$$c_{t+1}^{o,pd} = s_t^{y,pd}(1 + r_{t+1}) - \phi_{t+1} \tag{12}$$

Under the policy rule of constant per-capita debt as described in equation 10 one can solve equations 11 and 12 for the life-cycle budget constraint:

$$c_t^{y,pd} + \frac{c_{t+1}^{o,pd}}{1 + r_{t+1}} = w_t^y - b \cdot \frac{r_{t+1} - n_t}{1 + r_{t+1}}. \tag{13}$$

Comparing the life-cycle budget constraints given in equations 9 and 13 for Social Security and public debt respectively, the reader will immediately note the similarity between the two. The two alternative regimes will exactly be equivalent for $\tau = b$. Since b is defined as debt per capita of the old this translate into the case in which the retirement benefits to the old generation in t under Social Security must equal the size of next periods debt under the public debt regime in order for the two regimes to be equivalent.[7]

III.3 Equivalence in a macroeconomic context

The microeconomic equivalence will only hold if the equivalence is also true in a macroeconomic context. This is necessary since microeconomic equivalence will only hold under identical interest and wage rates. For the factor prices (w_t and r_t) to be equal under both schemes the per capita capital stock must be the same in both cases[8] which in turn requires macroeconomic equivalence.

To show the macroeconomic equivalence, I start out by stating the macroeconomic identities of (net)savings and investment under the alternative policies. From the respective identities one can derive the equilibrium conditions for the capital market in per capita terms.

First I look at the economy with Social Security in which private savings S^{ss} equal macroeconomic investment I^{ss}. Setting the rate of depreciation to zero, investment equals the change in capital stock: $I_t = K_{t+1} - K_t$. Private saving is the sum of savings by young households $S_t^{y,ss}$ and savings by old households $S_t^{o,ss}$. The old generation will not live to see another period so it will be optimal for them not to posses any assets after this period. Obviously the old dissave their entire savings which equals the existing capital stock K_t^{ss}. So the macroeconomic identity of savings and investment can be expressed as $S_t^{y,ss} = K_{t+1}^{ss}$. As the capital-intensity k is defined in per young

[7]This point can easily be verified by multiplying $\tau = b$ by N_t^y and using $\tau_t^y N_t^y = tr_t N_t^o$ and from the definition of public debt $B_{t+1} \equiv bN_{t+1}^o = bN_t^y$.

[8]The underlying assumption is a strictly concave production function.

capita the equilibrium condition for the capital market is described by:

$$s_t^{y,ss} = k_{t+1}^{ss}(1+n_t). \tag{14}$$

In an economy without Social Security, but with public debt and lump-sum taxation of the old generation, investment I^{pd} equals the savings of private households minus newly issued public debt in the size of the deficit: $I_t^{pd} = S_t^{pd} - D_t$. Again investment equals the change in capital stock and private savings is the difference of savings of the young and dissaving of the old. Since public deficit equals the change in public debt we have: $S_t^{y,pd} + S_t^{o,pd} = K_{t+1}^{pd} - K_t^{pd} + B_{t+1} - B_t$. As before the old dissave all of their assets which equal the current stock of capital plus public debt. Solving for per capita terms we have:[9]

$$s_t^{y,pd} = k_{t+1}^{pd}(1+n_t) + b_{t+1}. \tag{15}$$

In order to prove macroeconomic equivalence I will first conjecture that the levels of capital-intensity are the same under both regimes $k^{pd} = k^{ss}$ and then go on to show that for $\tau = b$ the per capita capital stocks really are identical.

For $k^{pd} = k^{ss}$ it immediately follows that under both regimes interest rates and wages are equal: $r^{pd} = r^{ss}$ and $w^{pd} = w^{ss}$. But then for $b = \tau$ the RHS of equations 9 and 13 are identical. Since income and prices are equal for both cases all one has to assume is transitivity of preferences to conclude that the consumptions paths and in particular consumption when old must be equal under Social Security and public debt: $c_{t+1}^{o,ss} = c_{t+1}^{o,pd}$. Inserting the period two budget equations given in equations 6 and 12, respectively, into the equality of period two consumption under the two schemes we get:

$$(1+r_{t+1})s_t^{y,ss} + tr_{t+1} = (1+r_{t+1})s_t^{y,pd} - \phi_{t+1}. \tag{16}$$

Using the already derived policy rules for contributions (equation 8) and taxation (equation 10) one obtains equation 17.

$$(1+r_{t+1})(s_t^{y,pd} - s_t^{y,ss}) = \tau(1+n_t) + (r_{t+1} - n_t)b \tag{17}$$

Substituting the capital market equilibrium conditions for savings given in equations

[9]Note that for the macroeconomic context the definition of public debt per old capita is somewhat unlucky since capital is defined per young capita. This is why $(1+n_t)$ appears only in conjunction with k_{t+1} and not with b_{t+1}. The reason for nevertheless using this definition, is that the microeconomic equivalence is much more evident in equations 9 and 13 when debt is defined per old capita.

14 and 15, respectively and rearranging one can solve for:

$$k_{t+1}^{pd} = k_{t+1}^{ss} + \frac{\tau - b}{1 + r_{t+1}}. \tag{18}$$

It is easy to see, that for $\tau = b$ capital-intensities are the same under public deficit with taxation of the old and Social Security.

Since both economies converge to a steady state the steady state capital intensities and therefore labor income and interest rates are the same and hence I have verified the initial assumption necessary to show microeconomic equivalence. I summarize this result in Proposition 2.

Proposition 2 *An economy with a Social Security scheme is identical to an economy with constant public debt and taxation of the old, if the volume of retirement benefits to the current old under the Social Security regime equals next period's debt under the public debt regime. In particular the effects on the life-cycle budget constraints of the households, on the capital intensities, and therefore on wages and interest rates are identical under the two schemes.*

The implications of this proposition for the political debate are far reaching and should always be taken into consideration when discussing Social Security reform. The most important implications that can be derived directly from Proposition 2 are: i) a higher (lower) tax on the old is fully equivalent to a cut (rise) in Social Security benefits and ii) a transition from Social Security to a funded system might be economically neutral if terminating Social Security is paid via issuance of government bonds.

IV The present value of the perpetual implicit tax of PAYG Social Security

In section II it was shown that the gap between market return and the return on contributions under PAYG Social Security can be interpreted as an implicit tax. It is frequently argued that the present value of all future implicit tax payments exactly equals the initial benefit payments to the old. Among others this point has been made by Stiglitz et al. (1997) on an intuitive level. A proof of this point can only be found in Sinn (1999). Here I will proof this statement in a very simple but illustrative way by arguing – as before – solely from the budget constraint of the individuals. I will also discuss the restrictions that need to apply, which has been neglected so far.

IV.1 A simple proof

An instructive way of proving the above statement is to restrict the analysis to a simple case: Assume a small open economy, with constant fertility $n_{t+i} = n, \forall i$ and constant world capital stock. The assumption of a small open economy allows us to neglect macroeconomic feedbacks of policy options on the capital intensity of the economy. Furthermore since by assumption world capital stock is constant over time the interest rate will not vary: $r_{t+i} = r, \forall i$.

From equation 9 one can see that a PAYG Social Security levies an implicit tax in the amount of $\tau\frac{r-n}{1+r}$ on each individual at birth. The present value of the perpetual implicit tax (PIT_t) at time t equals the sum of all future implicit tax payments discounted to date t:

$$PIT_t = \tau\frac{r-n}{1+r} \cdot N_t^y + \tau\frac{r-n}{1+r} \cdot \frac{N_{t+1}^y}{1+r} + ... = \tau\frac{r-n}{1+r} \cdot \sum_{i=0}^{\infty} \frac{N_{t+i}^y}{(1+r)^i} \tag{19}$$

In order to proof that the present value sum of the perpetual implicit tax will exactly equal the transfer payment to the current old generation, one has to show that $PIT_t = trN_t^o$ holds. This is done easily by applying the functional representation of the demographic process given in equation 1 and noting that the infinite geometric series over $q \equiv \frac{1+n}{1+r}$ can be expressed as $\sum_{i=0}^{\infty} q^i = \frac{1}{1-q}$ for $|q| < 1$. The assumption of dynamic efficiency ($r > n$) is once again crucial since otherwise $q \geq 1$. We can rewrite equation 19 as:

$$PIT_t = \tau\frac{r-n}{1+r} \cdot N_t^y \cdot \frac{1}{1-\frac{1+n}{1+r}} = \tau N_t^y. \tag{20}$$

From the no-debt-policy-rule of the Social Security authority we know that the contribution payments of the young generations equals the benefit payments to the current old generations and $PIT_t = trN_t^o$ is verified. So in fact, one can see by inspection that the present value of the perpetual implicit tax does not only equal the initial benefit payment at introduction of PAYG Social Security, but that at every point in time the transfer to the old generation of that period equals the present value of the perpetual implicit tax from that time on. We state these two results in Proposition 3.

Proposition 3 *In a small open economy the Social Security benefit payments to the current old generation equal exactly the present value of the implicit tax of mandatory Social Security contributions paid by all future generations. In particular the initial benefit payments to the old at introduction of the Social Security system equal the present value of the perpetual implicit tax payments.*

The parallels between Social Security and public debt are again obvious: In both policies the government lays a burden on future generations in order to give a windfall-profit to current generation(s). In the public debt case this burden lies in the taxes that have to be paid to service the constant public debt and in the Social Security case it is the implicit tax of a lower return.

Proposition 3 implies that in a small open economy the debate on funded versus unfunded pension systems should focus on questions of intergenerational distribution rather than on efficiency-arguments. In a world with strictly positive growth rates it might be argued, that a benevolent planer will want to distribute some of the future income increases to the current old generation by introducing PAYG Social Security.

However two problems arise with this argument. The first lies in restricting the analysis to a small open economy and the second in the implied optimal policy rule. A one-time introduction of PAYG Social Security generates a windfall-profit for the old generation at that time while it burdens all future generations. So what a benevolent planer would have to do, is design a slowly expanding Social Security scheme that gives every generation a small windfall-profit from expanding the transfer.[10] Of course such a policy will not be sustainable in the long run, since the possibility to raise the contribution rate is bounded by wage income ($\tau \leq w$). Also an "optimal policy" – if one exists – is subject to a number of unknown parameters. Obviously such an "optimal policy" is purely academic and only feasible in a highly simplified setting that rules out any complications such as incentives to work.

Coming back to the underlying assumptions it turns out that constant population growth and interest rate are not essential for the proof.[11] The important assumption however is the fixed capital stock due to the small open economy. Thereby a higher saving rate of a single country as a response to switching regime from unfunded to funded pension does not have an influence on the world capital stock and therefore domestic capital stock, income, interest rates, and wages are not altered by the countries choice of funding or not funding Social Security.

Two problems arise with this assumption. First, the degree of integration of world capital markets remains an open question. Two empirical relationships suggest that a small open economy might not be the correct framework. Feldstein and Horioka (1980), Frankel (1991), and Taylor (1996) have found a strong correlation between national saving and national investment rates. This implies that changes in the domestic saving rate do have an influence on the domestic investment rate and therefore on national

[10]This expanding-transfer-policy is equivalent to introducing another PAYG Social Security on top of the existing one. An excellent exposition of such benefit-increases can be found in Stiglitz et al. (1997).

[11]In his proof for the same point Sinn (1999) uses non-constant population growth and interest rates and only needs the assumption that an infinite sum similar to our sum over q converges. Even though it is not important that interest rates are not constant, it is important however whether the interest rate is determined endogenously (see below).

capital stock, national product, and factor prices of that country. The second empirical finding concerns the substantial ”home bias” in equity ownership that can be observed despite a large volume of cross-border capital movements. French and Poterba (1991) estimate that around ninety percent of equity assets of U.S. and Japanese investors are held in their domestic equity market.

The second problem with the assumption of a fixed world capital stock is that even if capital markets were so strongly integrated that the domestic saving rate had no impact on the domestic capital intensity this assumption would still break down, if a major part of industrialized countries were thinking of privatizing Social Security. If a substantial number of countries were to change their Social Security policy this again would have an effect on capital intensity. However in most OECD countries some sort of funding policy can be observed in the past years.[12]

IV.2 Dynamic Efficiency, Inefficiency, Social Security, and Bubbles

Last but not least we have to address the question of dynamic efficiency. The term dynamic efficiency characterizes an economy where the interest rate is greater or equal to the growth rate of the economy. If, on the other hand, the interest rate is smaller than the growth rate, one speaks of dynamic inefficiency for in such a situation it is possible to increase consumption of all generations, current living and future ones, by lowering the saving rate of the economy.[13] The theoretical backbone of dynamic efficiency and inefficiency is the neoclassical growth theory with its neoclassical production function and competitive factor markets where the interest rate is equal to the marginal product of capital. The decreasing marginal return of capital implied by a concave production function leads to this possible ”over-saving” of the economy in the dynamic inefficient region.

Obviously it is of crucial importance whether an economy is on a dynamic efficient or inefficient growth path, since in the case of dynamic inefficiency (efficiency) any government activity that decrease net-savings increases (decreases) the welfare of the economy. Of course both public debt and Social Security are devices to lower national saving and investment. Another implication for government policy in the dynamic

[12]Chile (1980), Australia (1991), Argentina (1993), and Mexico (1995) all practically replaced there PAYG system by a funded one. Most of the United Kingdoms occupational pension liabilities are already funded. In 2000 U.K. private-sector pension funds already had 600 billion pounds worth of investment (Budd and Campbell (1998)). According to Poterba et al. (2000) personal assets from the 401(k) pension plan in the U.S.A. will be substantially greater than Social Security plan wealth for persons retiring three decades from now. Germany has only very recently (January 2001) adopted a supplementary funded pillar to the existing PAYG Social Security. For an international overview of Social Security see Gruber and Wise (1999). Feldstein (1998) covers countries that have already implemented a shift toward funded systems.

[13]For the case in which $r = n$ ones speaks of the golden rule savings rate, since at that savings rate steady state consumption is maximized.

inefficient region is that when issuing new public debt the per capita debt can be reduced by just paying the interest rate and not actually paying back the principal debt, since the population is growing faster then the debt services.

Theoretically an economy should only be in the dynamic inefficient region if for one reason or another markets are missing that prevent Pareto-improving trade. One such an argument has been made for the missing possibility to trade between generations over time. In the highly simplified setting of the two period OLG economy the point becomes obvious: No two generations are both alive at two points in time. This missing market for trade between generations has been put forward first by Diamond (1965) as one of the arguments in favor of Social Security or public debt.

The discussion on whether economies can be or are in a steady state in the dynamic inefficient region remains mostly an empirical question. Still it has been neglected in large parts of the literature on Social Security that even if an economy were in this dynamic inefficient region Social Security or public debt are not necessarily the imperative response. As Tirole (1985) has shown that even for $r < n$ in the initial period there exists an asymptotically bubbly equilibrium that is efficient and converges to the golden rule steady state. The argument relies on the fact that asset bubbles can be part of a rational expectations equilibrium if the value of the bubbles grows at the rate of the interest rate. A bubble exist if the price of an asset exceeds the market fundamentals. The bubble serves as an substitute to investing in productive capital and therefore the capital intensity is reduced and interest rates rise over time converging to the respective golden rule levels.

The advantage of such a bubbly equlibrium[14] is that it converges to the golden rule steady state by the forces of markets and rational behavior whereas in the public debt or Social Security case the government will have to have knowledge on the correct level of public debt necessary to reach the golden rule steady state. On the other hand, bubbles can also be potential sources of inefficiencies due to costly bubble creation or nonexhaustion of resources.

Important for the discussion on Social Security is the point that even though most economies feel it is save to assume that dynamic efficiency prevails – as I have done throughout this paper – implementing a Social Security scheme might not even be necessarily the optimal policy even if the economy were initially in a dynamic inefficient state. Bubbles might do the job for the government.

[14]Note that national debt does not contain a bubble it only acts as one. See Tirole (1985) p. 1518.

IV.3 On the discussion of switching regimes

In the past three decades economists have made a case for switching the PAYG Social Security to a funded or at least partially funded system. The arguments for such a switch centered around i) the implicit tax of Social Security captured by the Aaron-Theorem and ii) the loss in aggregate savings and therefore investment due to the nature of a PAYG system. The first problem with both arguments is that their relevance is an empirical question. Argument i) only applies when we are in the dynamic efficient region (see discussion above) and argument ii) has been put in doubt by the so called Munell-Feldstein controversy. Economists have found empirical evidence for and against both arguments.[15] The major part of economists is however willing to assume that dynamic efficiency applies and that Social Security does in fact lower the national saving rate. Following this judgement one runs immediately into a new problem: Even though the new steady state of a funded system might be a clear welfare improvement in comparison to the current situation with an unfunded system one has to take account of the transition period and the generations that are alive during that period.

The question of whether Pareto-superior switching strategies, i.e. strategies that will make every generation better or at least as well off as before, are possible has been at the center of the debate on funded vs. unfunded Social Security at the beginning of the 1990s. A recent survey of the literature can be found in Hirte (2000). A number of authors have shown that efficiency gains are possible.[16] However in all cases the authors combine the effects of funding Social Security and changing the tax base or eliminating further distortions like early retirement incentives. The welfare gains from the latter policy change can of course always be realized separately without funding Social Security. Hirte makes the point that Pareto-superior switching strategies are not possible without also changing the tax base. So he analyses the political feasibility of changing to a (partially) funded system by looking for majorities and comes to the conclusion that (in the German case) the prevailing system is preferred by the majority of voters. One would have to install some compensating mechanism for the elderly and middle-aged in order to reach a majority.

V Intergenerational risk sharing and PAYG Social Security under uncertainty

Until now we have only considered economies without uncertainty about future outcomes, in particular the development of productivity and asset returns. With no un-

[15] Compare Börsch-Supan (2000) for a survey of recent empirical literature on the topic.
[16] See for example Breyer and Straub (1993) and Raffelhüschen (1993).

certainty about the future there is no need to insure oneself against possible bad realizations of future states of the economy. But in a world with uncertainty this is exactly what PAYG Social Security (and public debt) can provide: Insurance.

The argument can be outlined as followed: Consider a stochastic process for the long run development of labor-productivity and/or the return of capital. The length of a cycle is roughly equal to the length of a period in an overlapping generations model, e.g. 30 years. When young, a generation is exposed to the risk that the returns on their asset savings might be in good state (high return) or bad state (low return) when old. In general this generation will want to insure itself against a possible bad outcome. However there is no other generation alive to make such a contract with: The current old will be dead in the next period and the next generation is not born yet. When the next generation is born in $t+1$, there is no longer any uncertainty about periods $t+1$ outcome. After the realization of the state no insurance contract will be signed since such a contract at that point in time is not in the interest of one of the two parties. One can see that the argument is very similar to the missing markets argument of Diamond (1965): Without government intervention a contract that improves welfare is not feasible because no two generations are alive both at the time when the contract would have to be signed and at the time when the contract would have to be fulfilled. However in contrast to Diamonds case the inefficiency here is not due to non-optimal over-saving but to under-insurance. Only the government can implement an institution that enables the generations to share risk among each other.

This argument has first been made by Weiss (1979) focusing on the effects of money supply. An excellent exposition in a more general context can be found in Gordon and Varian (1988). Fisher (1983) and Gale (1990) analyze risk sharing implications of public debt, while Enders and Lapan (1982), Hansson and Stuart (1989), and Thøgersen (1998) consider intergenerational risk sharing in Social Security programs. Bohn (1997, 1998) compares public debt and Social Security as instruments to share risk between generations and comes to the conclusion that under uncertainty there is a substantial difference between public debt and Social Security in contrast to the equivalence under certainty derived earlier. In particular, he concludes that a market solution allocates too much productivity risk on the young and too little on the old. Moreover public debt shifts even more risk on the young, whereas Social Security is basically risk neutral.

To illustrate the argument I will follow Thøgersen (1998) and apply the uncertainty argument in a Social Security setting with a stochastic component to labor income. Assume an economy where the labor income of the young generation is composed of $w + \epsilon_t$, where ϵ_t is an independent identically distributed (i.i.d.) shock to productivity with an expectation of 0 and σ^2 variance. To keep things simple, we further assume that wage and interest rates are independent of the capital stock and that interest rate and population growth are both zero.[17] The period budget constraints (depicted P1 and P2 below) for the fully funded and PAYG Social Security are very similar to the earlier versions with $r = 0$ and a stochastic term ϵ_t. The only difference is that before the Social Security payment was a lump-sum payment τ and now $0 < \tau < 1$ is a fixed proportion of labor income.[18] Making the Social Security benefit a fixed proportion of the young generations labor income[19] will be the instrument that helps to share risk amongst generations.

The benefit payments in the period when old will then be the same fixed proportion τ of the next generations labor income when young. Benefit payments are given by the Social Security budget constraint: $tr_{t+1} = (w + \epsilon_{t+1}) \cdot \tau$.[20] The period budget constraints together with the Social Security budget constraint then yield the life-cycle budget-constraints (LC, last line below) under the two systems:

	funded	pay-as-you-go *SS*
P1	$c_t^{y,f} = w + \epsilon_t - s_t^{y,f}$	$c_t^{y,ss} = (w + \epsilon_t)(1 - \tau) - s_t^{y,ss}$
P2	$c_{t+1}^{o,f} = s_t^{y,f}$	$c_{t+1}^{o,ss} = s_t^{y,ss} + tr_{t+1}$
LC	$c_t^{y,f} + c^{o,f} = w + \epsilon_t$	$c_t^{y,ss} + c_{t+1}^{o,ss} = w + (1 - \tau)\epsilon_t + \tau\epsilon_{t+1}$

It is obvious that the expected value of life-cycle income before realization of ϵ_t is equal to w under both regime since $E[\epsilon_t] = 0, \forall t$. However the variance of life-cycle income differs. Under the fully funded scheme the variance of life-cycle income is equal to the variance of the productivity shock σ^2, while under the Social Security scheme the variance of life cycle income equals

$$Var[y_t^{LC,ss}] = (1 - \tau)^2 Var[\epsilon_t] + \tau^2 Var[\epsilon_{t+1}] + 2\tau(1 - \tau)Cov[\epsilon_t, \epsilon_{t+1}]. \tag{21}$$

Because in our simple example ϵ is distributed independent and identical the covariance of shocks between periods is zero and the variances of the shock is equal to σ^2 in every period. The variance under Social Security therefore equals $[(1 - \tau)^2 + \tau^2]\sigma^2$. The value $\tau = \frac{1}{2}$ minimizes this variance at $\frac{1}{2}\sigma^2$. The variance is reduced because the young share the outcome of their productivity shock with the current old and will in return participate in next periods productivity shock ϵ_{t+1}. It is obvious why the market allocation could not pool the exposure to productivity risk over time while a government institution can do so.

[17] This case is a special case of a golden rule steady state since $r = n$. For Social Security issues this specific golden rule steady state is equivalent to all other golden rule steady states.

[18] All earlier results remain unchanged in a setting with Social Security contributions proportionate to labor income.

[19] Thøgersen (1998) refers to this as *fixed tax rate.*

[20] Note that $L_{t+1}^o = L_{t+1}^y$ from the assumption of zero population growth.

In this setting Social Security will reduce the variance of life-cycle income and therefore increase welfare for risk-averse individuals. The unambiguous sign of the positive welfare change is of course somewhat constructed. As earlier discussed a Social Security scheme imposes an implicit tax on the individuals so that in a more realistic setting one would have to expect a reduction in expected life cycle income due to Social Security contributions. So for an analysis of welfare effects of Social Security one has to consider a trade-off between a welfare loss from the decrease in in life-cycle resources and a welfare gain from the decrease in life-cycle resource variance.[21] A policy recommendation will therefore require knowledge of the degree of risk aversion.

For a more realistic description of the risk-implications one would have to take three further complicating issues into consideration. First, not only labor income is stochastic but also the net-return of capital and even the value of capital itself may be stochastic. Both theory and empirical evidence point to highly correlated return of labor and capital when looking at longer time periods.[22] So when taking into consideration that the returns on savings when old are exposed to the same risk as wage rates of the next young generation, we see that the benefits from risk sharing are reduced.

Second, one has to question the assumption of independent shocks. Empirical evidence point much more to a highly correlated autoregressive process for productivity. But if the error terms are correlated, the covariance term in equation 21 will be positive and benefits from risk sharing are further reduced. In fact, with high serial correlation of the stochastic term in this very simple model the covariance is close to the value of the variance and the risk-sharing-benefits nearly disappear completely. Empirical evidence show that productivity growth is an i.i.d. stochastic process so that productivity levels are indeed a random walk (see Bohn (1998)).

Third, we have again neglected macroeconomic feedback effects that can only be captured in a dynamic general equilibrium framework. Intuitively one will expect that macroeconomic feedback will dampen the welfare increase of intergenerational risk sharing. A positive (negative) productivity shock in t will lead to an increase (decrease) in savings of the young because of the positive (negative) income effect. This will in

[21]Reintroducing non-zero interest and growth rates the no-debt constraint of Social Security implies policy rule $tr_{t+1} = \frac{1+n}{1+r}(w + \epsilon_{t+1})\tau$. Life-cycle resources will than equal $y_t^{LC,ss} = w - \tau w\frac{r-n}{1+r} + (1-\tau)\epsilon_t + \tau\frac{1+n}{1+r}\epsilon_{t+1}$ so that expected resources are less under a Social Security scheme than under a fully funded one: $E[y_t^{LC,ss}] = w - \frac{r-n}{1+r}\tau w < w$. The variance will however also be decreased: $Var[y_t^{LC,ss}] = \sigma^2[(1-\tau)^2 + (\frac{1+r}{1+n}\tau)^2]$.

[22]Empirical evidence in Baxter and Jermann (1997) show that at long periods factor incomes are almost perfectly correlated. From the theoretic side this is easily seen when assuming a standard Cobb-Douglas production function of the form $Y = A_t K_t^\alpha L_t^{1-\alpha}$. Since wage and interest rate are the respective derivatives $w_t^y = \frac{\partial Y}{\partial L} = (1-\alpha)A_t(\frac{K_t}{L_t})^\alpha$ and $r_t = \frac{\partial Y}{\partial K} = \alpha A_t(\frac{K_t}{L_t})^{\alpha-1}$. Obviously both are positively dependent to productivity shocks captured in A_t. Note that the specification here resembles a Hicks neutral technological progress. A capital augmenting or labor augmenting progress would lead to a greater difference in elasticities of the respective factor return to the corresponding type of progress.

turn lead to a higher (lower) capital intensity in period $t+1$. A higher (lower) capital intensity will lead to a lower (higher) interest rate and a higher (lower) wage rate in $t+1$. Hence we have two effects that reduce the need and effectiveness of insurance against productivity shocks. The first effect is that a positive shock will induce lower asset return in the next period via the macroeconomic feedback. The second effect is that the (deterministic part) of next periods wage rate is positively dependent on this periods shock which reduces the effectiveness of the insurance via payroll tax.

Understanding risk aspects in general equilibrium economies with production has only very recently become a topic of economic research. The problems of analyzing and understanding these effects are twofold. For one thing, as can be seen in this section, realistic models on this topic are a very complex problem. First successes in dealing with this problem have been notably made by Henning Bohn in a series of articles (see Bohn (1998,1999,2001)). The second problem lies in the restricted possibility of finding answers by looking at the data. Because we need to look at generational frequency, even excellent data sets reaching back as far as the 1870s only provide about five observations. Obviously, estimates can not be made at a significant level. The issue of risk aspects of different Social Security schemes will however be the essential question in the debate on to what degree and how Social Security should be privatized.

VI Conclusion

This paper provides a summary of what one should keep in mind when discussing unfunded versus funded Social Security. Instead of commenting one single countries recent and current ambitions on reforming Social Security, I show the most fundamental arguments in an analytical but still easy to follow way. In essence what one should bear in mind are four arguments:

First, there is an implicit tax on Social Security contributions because of the lower than market internal rate of return on PAYG Social Security contributions.

Second, depending on the exact policy, Social Security and public debt are equivalent or near equivalent. This is due to the fact that under a scheme that keeps per capita public debt constant the necessary taxes to pay interests on the existing debt equals the implicit tax of the Social Security contribution. The implication: Funding strategies of Social Security that are debt financed might actually be equivalent to the initial PAYG system.

Thirdly, one has to be careful about the efficiency gains from switching from PAYG to a unfunded scheme. One gain that can always be realized (even without switching systems) is eliminating falsely set incentives concerning early retirement or an inefficient choice of the tax base. However whether there are efficiency gains in terms of higher national product because of higher saving rates crucially depends on i) how much world capital markets are really integrated and ii) whether a single industrial country or all industrial countries are going to ”privatize” Social Security.

Finally, one has to keep in mind that PAYG Social Security can serve as an instrument to share risk between generations.

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