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Christoph Borgmann

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INSTITUT FÜR FINANZWISSENSCHAFT
DER ALBERT-LUDWIGS-UNIVERSITÄT FREIBURG IM BREISGAU

Prof. Dr. A. Oberhauser
Waldackerweg 14
79194 Gundelfingen

Prof. Dr. B. Raffelhüschen
Platz der Alten Synagoge 1
79098 Freiburg i. Br.

Prof. Dr. W. Ehrlicher
Beethovenstrasse 23
79100 Freiburg i. Br.

Prof. Dr. H.-H. Francke
Maximilianstrasse 15
79100 Freiburg i. Br.

Labor income risk, demographic risk, and the design of (wage-indexed) social security

Christoph Borgmann*
Albert–Ludwigs–Universität Freiburg

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Abstract

Pay-as-you-go pension programs can help to share risk amongst generations. While a wage-indexed pension program is best suited to share labor income risk, I show that the combination of stochastic labor income and stochastic population growth may reduce the possibilities for intergenerational risk sharing: Labor income risk can only be shared when individuals are also exposed to demographic risk. For demographic uncertainty the usual categorization of pension programs does not suffice. I therefore introduce policies on how the demographic uncertainty is transmitted via social security. An optimal demographic indexation is derived for a small open economy and a closed economy.

JEL classification: H55, J1
Keywords: intergenerational risk sharing – social security – demography

*Institut für Finanzwissenschaft I, Albert–Ludwigs–Universität Freiburg, D-79085 Freiburg, Germany (Fax: +49-761-203 2290; borgmann@vwl.uni-freiburg.de). The author wishes to thank Daniel Besendorfer, Oliver Ehrentraut, Karen Feist, Stefan Fetzer, Philip Jung, Reinar Lüdeke, Stefan Moog, Bernd Raffelhüschen, and the participants of the public finance research seminar held at the CESifo institute in Munich in June 2002 for valuable comments.

1 Introduction

The intergenerational risk sharing characteristics of social security are a crucial argument in the defense of pay-as-you-go (PAYG) pension programs in the ongoing debate on privatizing social security. With overlapping generations a market solution to share risks between generations is not feasible. Enders and Lapan (1982) argue that social security can be a substitute for this missing insurance market. Thøgersen (1998) has shown, that for stochastic labor income the design of pension programs plays a crucial role: Only PAYG pension programs with wage indexation are capable of sharing risks. In this paper I point out, that the gains of sharing labor income risks across generations are however bought at the cost of exposing life-cycle resources to risks associated with the uncertainty of the population growth rate, that would otherwise not be present in a small open economy. Generally, under the presence of social security, population growth uncertainty may have an impact on the risk exposure of life-cycle resources via two channels: Firstly, the relation between contributions and benefits is affected by deviations from the steady state old-age-dependency ratio[1]. Therefore, the return on the contributions paid into the pension program is uncertain. Secondly, the wage rate may change depending on cohort size. While the first effect will always occur, the second effect depends on whether the fertility shock will have an effect on the wage rate, as it is predicted by neoclassical growth theory for a closed economy. I concentrate on how uncertainty concerning population growth influences the variance of life-cycle resources from an ex-ante point of view.[2] The question addressed in this paper is not on how to respond to a baby-boom baby-bust scenario, but on how a PAYG pension scheme transmits demographic risk onto the life-cycle resources of the individuals. For this purpose, I will add a stochastic population growth rate to the simple two-period overlapping generations framework introduced by Gordon and Varian (1988) and applied to the design of social security with stochastic labor income by Thøgersen (1998).

Furthermore, I will restrict the analysis to pension schemes with wage-indexation, as only these are capable of intergenerational risk sharing. However, under stochastic population growth wage-indexation does not fully describe the design of the pension program. Instead one needs to introduce policies on how the pension scheme reacts to a demographic (here: fertility) shock. I call these different possible pension policies *defined contribution wage indexation* (DC) and *defined benefit wage indexation* (DB). The basic difference between the two schemes is whether contributions (former) or benefits (latter) are adjusted in response to the realization of the uncertain population growth rate.

In the next section, I present the model and the two pension policies. In section 3, the risk sharing properties of the policy schemes are analyzed in a small open economy. In order to capture general equilibrium effects of demographic changes on labor income

[1] The old-age-dependency ratio describes the relationship between the number of retirees and the size of the potential work force.

[2] See Rangel and Zeckhauser (2001) on ex-ante versus interim optimality.

that should occur in a closed economy, I allow for the wage rate to change in response to fertility shocks in section 4. In section 5, I introduce a hybrid policy concerning the demographic indexation of the pension scheme and derive optimal demographic indexation rules for the small open and the closed economy. Section 6 concludes.

2 A simple overlapping generations model with stochastic labor income and stochastic population growth

I model risky labor income in a two period overlapping generations economy with a stochastic population growth rate under the presence of a PAYG pension scheme. In each period t there are two generations alive. The young generation inelastically supplies one unit of labor. The stochastic gross labor income is described by:

$$w_t = w \cdot \varepsilon_t, \tag{1}$$

where w is deterministic and ε_t is a normal independent identically distributed (n.i.i.d.) stochastic variable with mean one and variance σ_ε^2. From this gross labor income the individual will need to finance his youth consumption, a contribution to the pension scheme (τ_t), and further private savings for retirement. The real rate of interest (r) is exogenous, positive, and by assumption constant over time. When old, the individual does not work but consumes the gross return of his savings plus the social security transfer (tr_{t+1}). There is neither lifetime-uncertainty nor any bequest motive. The present value of life-cycle resources at birth (y_t) of a representative individual born in period t equal:

$$y_t = w_t - \tau_t + \frac{tr_{t+1}}{1+r}. \tag{2}$$

In order to capture risk aversion of the individuals in a simplified setting, I follow Gordon and Varian (1988) and Thøgersen (1998) and assume a mean-variance utility function, where utility increases with the expected present value of life-cycle resources but decreases with its variance:

$$U_t = u(\mathrm{E}[y_t]) - v(\mathrm{Var}[y_t]). \tag{3}$$

As a second source of uncertainty, I add a stochastic component η_t to the population growth rate. Since I am interested in the ex-ante risk implications of social security, I do not model a specific baby–boom baby–bust scenario. Instead, I introduce an additive stochastic component to the otherwise constant population growth rate. The demographic process is described by:

$$N_{t+1} = (1 + n + \eta_t)N_t, \tag{4}$$

where n is the deterministic part of the growth rate and η_t is a n.i.i.d. stochastic variable with mean zero and variance σ_η^2.[3]

For PAYG social security, I will only consider pension schemes that are indexed to labor income such as the *fixed tax rate* case of Thøgersen (1998). However, I will differentiate between a defined contribution and a defined benefit wage indexation.[4] I speak of a defined contribution system, when the contribution payments of the young generation are a fixed share γ of their income. The budget of the pension program is assumed to be balanced every period without running deficits or surpluses. Per-capita contribution and transfer payments for a representative member of generation t are therefore given by:

$$DC: \quad \tau_t = \gamma[w\varepsilon_t] \quad \text{and} \quad tr_{t+1} = \gamma(1+n+\eta_t)[w\varepsilon_{t+1}]. \tag{5}$$

In contrast, I speak of a defined benefit scheme when the retirees obtain a fixed share (replacement rate) ψ of the per-young-capita labor income. Here the contribution payment will vary with population growth in order to guarantee the promised pension payment. The respective per-capita contribution and transfer payments are then:

$$DB: \quad \tau_t = \frac{\psi}{1+n+\eta_{t-1}} w\varepsilon_t \quad \text{and} \quad tr_{t+1} = \psi w\varepsilon_{t+1}. \tag{6}$$

The difference between the two schemes lies in the exposure to the demographic shock: While in the defined benefit scheme the income of the old generation is independent of the realization of the demographic shock η_t, it is dependent on η_t in the defined contribution scheme. However, the respective contribution rates react just in the opposite way: Contribution payments of the young generation are adjusted under the defined benefit scheme, while they are held constant under defined contribution. Over the life-cycle the two schemes differ in respect to whether generation t is exposed to the demographic stochastic variable of period t (defined contribution) or to the demographic shock of period $t-1$ (defined benefit). Under both social security schemes the life-cycle resources (y_t) of the generation born in period t are subject to three realizations of stochastic variables: the realizations of the stochastic productivity term ε in t and $t+1$ and one realization of the stochastic population growth component η. I will now analyze the risk sharing characteristics of the different social security schemes.

[3] A normal distribution is somewhat problematic since it does not guarantee, that $(1+n+\eta) > 0$ and $w\varepsilon > 0$. Although a truncated normal could guarantee these conditions, an analytical derivation of the variance using such a distribution would be close to impossible. Restricting the variances of ε and η to values well below one will at least make such an event highly unlikely.

[4] Most authors do not look at a defined benefit wage indexation, but at a defined benefit scheme that is independent of the next generations income. See Hassler and Lindbeck (1998) for an intergenerational risk sharing analysis of a fixed tax rate with indexation versus a defined benefit scheme without indexation.

3 PAYG pension programs in a small open economy with stochastic labor income and population growth

In this section I focus the attention on a small open economy (SMOPEC). In a small open economy domestic per-capita capital stock is independent of domestic savings. This assumption together with perfect competition on factor markets assures that wage and interest rates will be constant. Therefore, macroeconomic feedback effects on factor prices are not present.

3.1 Defined contribution income indexation

I start out with the defined contribution scheme. In order to calculate the mean and variance of life-cycle resources I substitute τ_t and tr_{t+1} from equation 5 and $w_t = w\varepsilon_t$ into equation 2:

$$y_t^{DC} = w\varepsilon_t(1-\gamma) + \frac{\gamma w}{1+r}\varepsilon_{t+1}(1+n+\eta_t). \tag{7}$$

Because ε_{t+1} and η_t are independent, the expectation of equation 7 is equal to $\mathrm{E}[y^{DC}] = w - \gamma w \frac{r-n}{1+r}$. In comparison to a purely funded scheme, where $\tau = tr = 0$, the expectation of life-cycle resources $\mathrm{E}[y_t]$ will be smaller (larger) under PAYG social security than under a funded system when the economy is on a dynamically efficient (dynamically inefficient) growth path.[5]

The variance of y^{DC} is given by:[6]

$$\begin{aligned} \mathrm{Var}[y^{DC}] = w^2 \Bigg\{ & \left[(1-\gamma)^2 + \gamma^2 \left(\frac{1+n}{1+r}\right)^2\right] \sigma_\varepsilon^2 \\ & + \left(\frac{\gamma}{1+r}\right)^2 \sigma_\eta^2 + \left(\frac{\gamma}{1+r}\right)^2 \sigma_\varepsilon^2 \sigma_\eta^2 \Bigg\}. \end{aligned} \tag{8}$$

The term on the first line on the RHS of equation 8 is the *Thøgersen case* for a multiplicative labor income shock under deterministic population growth ($\sigma_\eta^2 = 0$). An indexed pension scheme with contribution rate $\gamma > 0$ reduces the variance of life-cycle resources in comparison to a fully funded scheme ($\gamma = 0$) where the variance is equal to $w\sigma_\varepsilon^2$. The second line shows the effect of stochastic population growth ($\sigma_\eta^2 > 0$): The *demographic-risk-effect* displays how much the variance of life-cycle resources rises due to demographic uncertainty added by the pension scheme. The first term of this line is the pure demographic risk effect while the second term is a combined uncertainty of both population growth and labor income. Both terms on line two are obviously positive for

[5] For a collection of fundamental results on social security see Diamond (1997), Sinn (2000), Feldstein and Liebman (2001), and Borgmann (2001).

[6] The variance is derived in appendix A.1 for the more general case with macroeconomic feedback effects discussed in section 4. Equation 8 is a special case of equation 19 with $\alpha = 0$.

$\gamma > 0$ and therefore lead to an increase of the variance. Since the demographic shock enters life-cycle resources in the second period, both terms on line two are discounted by $(1+r)^{-2}$. Note that line two is zero for an economy with deterministic population growth. The deviation of both lines together from $w\sigma_{\varepsilon}^2$ represent the total risk-impact of wage indexed social security with stochastic population growth and stochastic labor income in a SMOPEC. As one can see the variance unambiguously increases in comparison to the *Thøgersen case*. This leads immediately to the most important result: One cannot share labor income risk without being exposed to a demographic risk.

I analyze optimal policy in a sense that the optimal choice of γ is defined as the value of the tax rate γ^* that minimizes the variance of y_t^{DC} ($\gamma^* \equiv \underset{\gamma}{\operatorname{argmin}}\{\operatorname{Var}[y_t^{DC}]\}$). Differentiating equation 8 with respect to γ and solving for γ yields:

$$\gamma^* = \frac{1}{1 + \left(\frac{1+n}{1+r}\right)^2 + \frac{\sigma_\eta^2 + \sigma_\eta^2/\sigma_\varepsilon^2}{(1+r)^2}}. \tag{9}$$

In the golden rule steady state ($r = n$) we have $\gamma^* = \frac{1}{2+(\sigma_\eta^2+\sigma_\eta^2/\sigma_\varepsilon^2)(1+r)^{-2}}$, so that the optimal value of the tax rate for this special case is smaller than $\frac{1}{2}$, which would equal the Thøgersen solution for $r = n$.[7] The power of social security to share risk amongst generations is reduced when population growth is stochastic. Obviously, the optimal tax rate will have to be lower. How large this reduction will be, depends on the interest rate and the variances of the two stochastic variables. A higher interest rate will be associated with a larger optimal tax rate. The same can be said for the variance of labor income. A large variance of population growth will however reduce the optimal tax rate. This relationship becomes even stronger, if the variance of population growth is large in relation to the variance of labor income. Again, this is no surprise: Labor income shocks can only be shared between generations when life-cycle income is exposed to population risks that are otherwise not present in a small open economy. Hence, when the necessary evil of the remedy (population risk) is large relative to the initial flaw (labor income risk), the gains of a treatment (PAYG social security) are substantially lowered.

3.2 Defined benefit income indexation

Proceeding as before, life-cycle resources can be derived by substituting the social security policy rule given in equation 6 and labor income given in equation 1 into equation 2:

$$y_t^{DB} = w\varepsilon_t \left(1 - \frac{\psi}{1+n+\eta_{t-1}}\right) + \frac{\psi w}{1+r}\varepsilon_{t+1}. \tag{10}$$

Since the stochastic variable η is now part of the denominator the exact distribution

[7] The Thøgersen solution is obtained by setting σ_η in equation 9 equal to zero: $\gamma^*_{\text{Thøgersen}} = \frac{1}{1+\left(\frac{1+n}{1+r}\right)^2}$.

of y^{DB} cannot be determined. However we can derive some general conclusions about the expectation of y^{DB} and we can approximate both the expectation and variance for our special case with normal distributed population growth. The expectation of life-cycle resources is represented by:

$$\mathrm{E}[y_t^{DB}] = w + \frac{\psi w}{1+r} + \psi w \, \mathrm{E}\left[-\frac{1}{1+n+\eta_{t-1}}\right]. \tag{11}$$

Because $\frac{-1}{1+n+\eta}$ is strictly concave for $\eta > -(1+n)$ we have from Jensen's inequality that $\frac{-1}{1+n+\mathrm{E}[\eta]} > \mathrm{E}[\frac{-1}{1+n+\eta}]$ unless η is a constant with probability one. Therefore, the expectation of life-cycle resources under defined benefit is smaller for stochastic population growth than for deterministic population growth ($\mathrm{E}[\eta] = 0$ with probability one). Note that this is not the case under DC. For a comparison between the two schemes, I define an *equivalent certain benefit level* $\hat{\psi} \equiv \gamma(1+n)$. This is the benefit level that makes the defined benefit scheme equivalent to the defined contribution scheme under deterministic population growth.[8] Substituting this benefit level into equation 11 yields an important result: For an equivalent certain benefit level the expectation of life-cycle resources under DB is always lower than the expectation of life-cycle resources under DC: $\mathrm{E}[y_{\hat{\psi}}^{DB}] < \mathrm{E}[y^{DC}]$. Note that this result does not depend on the assumed distribution of the random variable η, but holds in general. It is solely due to the fact that under DC the life-cycle resources are a linear function of the population random variable, while under DB life-cycle resources are a concave function of the population shock.

For our specific case with an assumed normal distribution of η only an approximate solution can be derived. The quadratic approximation of $(1+n+\eta)^{-1}$ around $\mathrm{E}[\eta] = 0$ is given by:

$$\frac{1}{1+n+\eta} \approx \overbrace{\underbrace{\frac{1}{1+n} - \left(\frac{1}{1+n}\right)^2 \eta}_{\text{linear approximation}} + \left(\frac{1}{1+n}\right)^3 \eta^2}^{\text{quadratic approximation}}. \tag{12}$$

Substituting the quadratic approximation into life-cycle resources and taking the expectation yields:

$$\mathrm{E}[y^{DB}] \approx w\left(1 - \frac{\psi}{1+n}\right) + \frac{\psi w}{1+r} - \psi w \left(\frac{1}{1+n}\right)^3 \sigma_\eta^2. \tag{13}$$

The impact of social security on the expectation of life-cycle resources is twofold. Firstly, as always, social security changes life-cycle resources by $\frac{(n-r)\psi w}{(1+n)(1+r)}$ in comparison to a funded system. Secondly, for the case of defined benefit wage-indexation, the expected value of life-cycle resources is further reduced by $\psi w(1+n)^{-3}\sigma_\eta^2$. The size of this re-

[8] That the two pension policies react differently to the population shock is not touched by using this specific benefit level. It is only necessary to make the schemes comparable since the parameter determining the size of the pension program γ and ψ refer to different generations.

duction is a positive function of the replacement rate (ψ) and the variance of population growth (σ_η^2), but a negative function of the expected population growth rate $(1+n)$. In comparison, neither a funded system nor the defined contribution wage-indexation are subject to this second effect.

3.2.1 Linear approximation of the variance under defined benefit:

I start out to discuss the variance of y^{DB} with the linear approximation of $(1+n+\eta)^{-1} \approx (1+n)^{-1} - (1+n)^{-2}\eta$. Using a linear approximation to derive moments is often called the delta method. Substituting the linear approximation of $(1+n+\eta)^{-1}$ into the definition of the variance yields equation 14. I add the superscript ^{la} to denote the linear approximation:

$$\begin{aligned}\mathrm{Var}^{la}[y^{DB}] =& w^2 \left\{ \left[\left(1 - \frac{\psi}{1+n}\right)^2 + \left(\frac{\psi}{1+r}\right)^2 \right] \sigma_\varepsilon^2 \right. \\ & \left. + \frac{\psi^2}{(1+n)^4}\sigma_\eta^2 + \frac{\psi^2}{(1+n)^4}\sigma_\eta^2\sigma_\varepsilon^2 \right\}.\end{aligned} \tag{14}$$

Comparing the linear approximation of the variance for the DB case with the variance of DC at the certain equivalent benefit level ($\hat{\psi}$) shows the similarity of the two. For this benefit level the respective first line in equations 8 and 14 is equal, indicating that the pure productivity shock is shared identically within the two schemes. This feature will hold in general, independent of the order of the approximation. Inspection of the second lines of the respective equations shows, that there is only one difference between the two schemes: For DB we have a different "discounting mechanism" of the demographic risk than for DC. Remember that under DC the population risk components in the variance are discounted by the gross interest rate because the shock occurs in the second period. For DB the exposure to the shock is in the first period and therefore it is not really discounted. The convexity of the function $(1+n+\eta)^{-1}$ however leads to a "discounting mechanism", where the gross population growth acts as a demographic discount factor. This leads to the result that for $r > n$ (dynamic efficiency) the risk exposure is smaller under DC than under DB. For $n > r$ (dynamic inefficiency) the result is reversed.

3.2.2 Quadratic approximation of the variance under defined benefit:

Although easier to handle, using the linear approximation for $(1+n+\eta)^{-1}$ is somehow problematic because the delta method assumes that $\mathrm{E}[g(\eta)] = g(\mathrm{E}[\eta])$ holds asymptotically, so that $\mathrm{E}^{la}[y^{DB}] = \mathrm{E}[y^{DC}]$. The result derived in section 3.2 above concerning the expectation of y^{DB} does not emerge using the linear approximation. In order to ensure that the linear approximation does not overlook important features concerning the variance of y^{DB}, I compare the linear approximation with the quadratic approximation.

The variance of y^{DB} with the quadratic approximation of $(1+n+\eta)^{-1}$ given in equation 12 is derived in appendix A.2. I adopt the superscript ^{qa} to denote the quadratic approximation. The difference between the linear approximation and the quadratic approximation with $\psi = (1+n)\gamma$ equals:

$$\mathrm{Var}^{la}[y^{DB}] - \mathrm{Var}^{qa}[y^{DB}] = w^2 \left[2\frac{\gamma(1-\gamma)}{(1+n)^2}\sigma_\eta^2\sigma_\varepsilon^2 - \frac{\gamma^2}{(1+n)^4}(2\sigma_\eta^2 + 3\sigma_\eta^2\sigma_\varepsilon^2) \right]. \qquad (15)$$

This terms will be positive for $[\frac{1-\gamma}{\gamma}(1+n)^2 - 3/2]\sigma_\varepsilon^2 > 1$. If this condition holds, the linear approximation overestimates the variance and the risk sharing properties under DB are better than indicated in section 3.2.1. This will usually be the case for small γ and not too small values for σ_ε^2 and n. For larger γ and both small σ_ε^2 and n the difference given in equation 15 becomes smaller and the sign may even change.

Unfortunately the exact results of comparing risk aspects of DC versus DB using the quadratic approximation depends on the parameter values and on the question of whether the economy is in a dynamically efficient region.[9] Specifying parameter values does not seem like a fruitful way to determine a definitive answer to this question, since the model is highly stylized to begin with. Also, a more precise way of modelling risk aversion would be necessary in order to address the issue on how much the individuals are willing to give up in expected income for a reduction of the variance. What this section has however shown, is that a) the expectation of life-cycle resources will be lower under DB than under DC and that b) the discounting mechanism of the demographic shocks in the variance of life-cycle resources differs between the two schemes. All in all, from an ex-ante point of view in a small open economy and for $r > n$, a waged indexed PAYG pension scheme with defined contribution seems to be superior to a defined benefit scheme, if the level of social security (γ) is not close to zero.

4 Defined contribution and defined benefits in a closed economy

In this section, I expand the analysis by making the wage rate dependent on the cohort size of the working generation. Because I want to concentrate on stochastic labor income and population growth, I keep the assumption of a constant interest rate. Even though this procedure is far from modeling a general equilibrium economy, it does however capture the effects of fertility shocks on the wage rate as one would expect to see them in a simple general equilibrium model of a closed economy. Specifically, I assume that

[9] For $r = n$ (golden rule) the difference given on the RHS of equation 15 is also equal to the difference of $\mathrm{Var}[y^{DC}_{r=n}] - \mathrm{Var}^{qa}[y^{DB}_{\hat{\psi},r=n}]$. So for the variance to be smaller under DB than under DC the same conditions apply as for the sign of the difference between the linear and the quadratic approximation.

the wage rate of period t is given by:

$$w_t = w(\varepsilon_t - \alpha\eta_{t-1}). \tag{16}$$

To see that this specification replicates the macroeconomic effect of population growth on labor income, note that η_{t-1} is the shock that determines the size of the population born and working in period t. The population N_t is larger for a positive realization of η_{t-1}. From a macroeconomic perspective one should expect the wage rate to decrease for larger cohort-sizes. The parameter α determines the size of the feedback effect and can be interpreted as the relative change of the wage rate near its steady state value given an absolute change in population growth of size η_{t-1} or in other words as the scaled population-growth-elasticity of the wage rate.[10]

In the remainder of this section, I will first investigate the expectations of life-cycle resources under both schemes. Then I will look at the variance of y^{DC} with $\alpha > 0$ and show how the introduction of the macroeconomic feedback effect changes the results of section 3. A comparison of the variance of life-cycle resources under defined contribution and defined benefit concludes the section.

4.1 Life-cycle resources under defined contribution and defined benefit

By substituting the contribution and benefit payments given in equations 5 and 6 for the pension policies and labor income from equation 16 into equation 2, one can derive the life-cycle resources under DC and DB respectively:

$$\begin{aligned} y_t^{DC} &= w(\varepsilon_t - \alpha\eta_{t-1})(1-\gamma) + \frac{w\gamma}{1+r}(\varepsilon_{t+1} - \alpha\eta_t)(1+n+\eta_t), \\ y_t^{DB} &= w(\varepsilon_t - \alpha\eta_{t-1})\left(1 - \frac{\psi}{1+n+\eta_{t-1}}\right) + \frac{w\psi}{1+r}(\varepsilon_{t+1} - \alpha\eta_t). \end{aligned} \tag{17}$$

Taking the expectation of life-cycle resources under the two respective policies given in equation 17 yields:

$$\begin{aligned} \mathrm{E}[y^{DC}] &= w(1-\gamma) + w\gamma\frac{1+n}{1+r} - w\alpha\frac{\gamma}{1+r}\sigma_\eta^2, \\ \mathrm{E}[y^{DB}] &\approx w(1 - \frac{\psi}{1+n}) + \frac{w\psi}{1+r} - \frac{w\psi}{(1+n)^3}\sigma_\eta^2 - w\alpha\frac{\psi}{(1+n)^2}\sigma_\eta^2. \end{aligned} \tag{18}$$

[10] To see that this ad-hoc specification bears some economic reason, note that the wage rate given by a Cobb-Douglas Production function with competitive factor markets is given by $w = (1-\tilde{\alpha})(K_t/N_t)^{\tilde{\alpha}}$, where $\tilde{\alpha}$ is the labor share of per-capita income and K_t is the aggregate capital stock in period t. Substituting $N_{t-1}(1+n+\eta_{t-1})$ for N_t and linearizing around the steady state (only considering changes in population growth) yields: $\frac{dw}{w_{ss}} = -\tilde{\alpha}\frac{d(1+n)}{(1+n)_{ss}}$, so that $-\tilde{\alpha}$ can be interpreted as the population-growth-elasticity of the wage rate. In the specification of labor income in equation 16 we are however considering absolute changes of the population growth rate instead of relative changes. So in order to be precise, one should note that $\alpha = \tilde{\alpha}/(1+n)$.

In comparison to the SMOPEC (section 3) the expectations for both policies are reduced by the respective last term. The size of this term depends on the size of the feedback effect, the size of social security, the variance of population growth, and a "discount" factor. Again the "discounting" of the terms associated with σ_η^2 differs: For equivalent certain benefit level $\hat{\psi}$, the discount factor is $(1+n)^{-1}$ for DB, while it equals $(1+r)^{-1}$ for DC.

4.2 Variance of life-cycle resources under defined contribution

The variance of life-cycle resources under DC with macroeconomic feedback is given in equation 19 (see appendix A.1):

$$
\begin{aligned}
\mathrm{Var}[y^{DC}] = w^2 \Bigg\{ & \left[(1-\gamma)^2 + \gamma^2 \left(\frac{1+n}{1+r} \right)^2 \right] (\sigma_\varepsilon^2 + \alpha^2 \sigma_n^2) \\
& + \left(\frac{\gamma}{1+r} \right)^2 (1+\sigma_\varepsilon^2)\sigma_\eta^2 \\
& + 2 \left(\frac{\gamma}{1+r} \right)^2 \alpha^2 \sigma_\eta^4 - 2 \left(\frac{\gamma}{1+r} \right)^2 (1+n)\alpha\sigma_\eta^2 \Bigg\}.
\end{aligned}
\tag{19}
$$

There are important differences for the variance under DC in the closed economy in comparison to the SMOPEC: The term in square brackets on the first line of equation 19 is now multiplied by the variance of wage income (σ_ε^2) plus the variance of population growth (σ_η^2) times the coefficient for the macroeconomic feedback squared (α^2). This indicates, that in a closed economy uncertain population growth will already have an impact on the variance of life-cycle resources in a fully funded system. In particular, the variance of life-cycle resources in a fully funded system equals the variance of labor income: $\mathrm{Var}[w_t] = (\sigma_\varepsilon^2 + \alpha^2\sigma_n^2)w^2$ (equation 19 with $\gamma = 0$). Because w_t is dependent on demographics, labor income underlies the risk of uncertain demographic developments. Therefore, social security reduces the risk of fluctuating labor income due to both the productivity shock and the demographic shock. So in the closed economy the benefits of sharing the risk of fluctuating wage income over generations are greater than in the SMOPEC. However, the risk of fluctuating wage income due to fertility shocks cannot be shared as easily as the productivity uncertainty. The necessary evil of fluctuating benefit payments adds risk to life-cycle resources. Specifically, the increase of the variance is of size $w^2(\frac{\gamma}{1+r})^2(1+\sigma_\varepsilon^2)\sigma_\eta^2$. This effect is identical for the SMOPEC and the closed economy.

The second difference between the SMOPEC and the closed economy is observable in the last line of equation 19. This line can be interpreted as a covariance between wage income and the replacement rate in period $t+1$: The same shock η_t has an influence on both the replacement rate in $t+1$ and the wage income of $t+1$. Since the two effects are of opposite directions this "covariance" is negative.[11]

The argument, that a PAYG pension program helps to share risks between generations in an economy where demographic fluctuations have a strong impact on factor prices, has been made in defense of sustaining a PAYG scheme during a baby bust scenario by Smith (1982) and Bütler and Harms (2001): While the PAYG scheme is per se exposed to demographic risks the welfare of the different generations during this demographic transition is affected inversely by macroeconomic effects on factor income. Bohn (2001) and Young (2001) also come to the conclusion that large generations are usually hit hardest because of general equilibrium effects on factor returns.[12]

4.3 Defined benefit versus defined contribution in a closed economy

For the sake of a tractable representation, I will only discuss the variance of life-cycle resources under DB using the linear approximation for $(1+n+\eta_{t-1})^{-1}$. The variance of y^{DB} using the quadratic approximation is given in appendix A.3. Also, I will concentrate on a comparison of DB versus DC, since the general direction of the results are similar for DC and DB. There will be, however, one important new difference between DB and DC when considering macroeconomic feedback on the wage rate: Because of the different timing in the pension schemes DB will offer better insurance for $t-1$ fertility shocks, while DC offers better insurance for η_t.

Substituting the linear approximation for $(1+n+\eta)^{-1}$ from equation 12 into y^{DB} given in equation 17 yields the following variance:

$$\begin{aligned}\mathrm{Var}^{la}[y^{DB}] =& w^2\Bigg\{\left[\left(1-\frac{\psi}{1+n}\right)^2+\left(\frac{\psi}{1+r}\right)^2\right](\sigma_\varepsilon^2+\alpha^2\sigma_\eta^2)\\ &+\frac{\psi^2}{(1+n)^4}(1+\sigma_\varepsilon^2)\sigma_\eta^2 \\ &+2\frac{\psi^2}{(1+n)^4}\alpha^2\sigma_\eta^4-2\frac{\psi}{(1+n)^2}\left(1-\frac{\psi}{1+n}\right)\alpha\sigma_\eta^2\Bigg\}.\end{aligned} \tag{20}$$

The difference between the SMOPEC and the closed economy under DB is similar to that difference under DC: With the macroeconomic feedback effect wage income is more risky than without this effect and the gains of insurance via social security are greater (first line). The risk of an uncertain return on the contributions paid into the social security scheme is the same with or without a feedback effect and increases the variance

[11] To verify this, note that for plausible values $1+n>\alpha\sigma_\eta^2$, because α and σ_η^2 both should be well below one, so that n must only be not too much below zero to guarantee this condition.

[12] The macroeconomic feedback argument is even stronger when including capital accumulation and uncertain or endogenous asset returns. See Poterba (2001) and Brooks (2000, 2002) on demographics and asset returns.

(second line). Finally, as under DC, there is also a "covariance term" that will reduce the variance (third line). However, there is a difference between the respective "covariance terms" for DB and DC.

Since the discounting mechanism of the population shock in the variance still differs between DB and DC, I compare the two schemes in a golden rule steady state with replacement rate $\hat{\psi}$. For this specific case, the variances of the two policies only differ in the last term on the third line. Subtracting the variance of y^{DB} given in equation 20 with $\psi = \hat{\psi}$ and $r = n$ from the variance of y^{DC} given in equation 19 yields:

$$\mathrm{Var}[y^{DC}_{r=n}] - \mathrm{Var}^{la}[y^{DB}_{\hat{\psi},r=n}] = \frac{2\gamma}{1+n}(1-2\gamma)\alpha\sigma^2_\eta. \tag{21}$$

From equation 21 one can see, that for $\gamma < 0.5$ risk sharing is more efficient under DB than under DC.

To clarify this further, I show the influence of positive population shocks in periods $t-1$ and t on the life-cycle resources of generation t under the two policies in a more general setting in table 1. The positive fertility shocks of both periods negatively affect labor income in t and $t+1$, respectively. However, under DC the positive shock η_t will have a positive influence on the return from the pension program. Under DB this will be the case for a positive realization of random variable η_{t-1}. The influence of the fertility shock on the return from social security is always of opposite direction from the influence of the same shock on labor income itself.

Table 1: Effects of positive fertility shocks on life-cycle resources

DC	$w(\varepsilon_t, \eta_{t-1}) \cdot$	$(1-\gamma)$	$+$	$\frac{\gamma(1+n+\eta_t)}{1+r}$	$\cdot\, w(\varepsilon_{t+1}, \eta_t)$
$\eta_{t-1} > 0$	$-$				
$\eta_t > 0$				$+$	$-$
DB	$w(\varepsilon_t, \eta_{t-1}) \cdot$	$(1-\frac{\psi}{1+n+\eta_{t-1}})$	$+$	$\frac{\psi}{1+r}$	$\cdot\, w(\varepsilon_{t+1}, \eta_t)$
$\eta_{t-1} > 0$	$-$	$+$			
$\eta_t > 0$					$-$

In general, social security ensures that each generation participates in both fertility shocks and therefore helps to spread risk across generations (first line in equations 19 and 20). However, the "covariance term" (third line in equations 19 and 20) will only apply for one of the shocks in each scheme: For DC this is the shock η_t, while for DB it is η_{t-1}. But η_{t-1} is the shock that influences the labor income earned by generation t when young and $(1-\gamma)$ roughly equals the weight of a generation's own labor income in their life-cycle resources. For $\gamma < 0.5$ the influence of a generation's own labor income dominates that generation's life-cycle resources. It follows, that in a realistic setting, where $\gamma < 0.5$, DB offers better insurance than DC because DB provides insurance against fluctuations of the random variable η_{t-1}, which will have a greater weight on overall life-cycle resources of generation t.

All in all, in a closed economy social security leads to a reduction in the expectation of life-cycle resources that is independent of dynamic efficiency or dynamic inefficiency. But at the same time PAYG pension programs help to insure two risky components of labor income across generations: Productivity shock and fertility shocks. Productivity risk is always reduced via social security. The overall influence of fertility shocks on the variance of life-cycle resources depends in sign and size on the size of the macroeconomic feedback effect and the variance of the population growth rate. Whether social security absorbs or adds risk due to uncertain population growth, depends on how strong the effects of demographic changes are on future factor incomes. The prediction of a dynamic general equilibrium model of a closed economy with a Cobb-Douglas production function is, that demographic risks are reduced by social security. The assumptions underlying this framework have however been put into question (see the discussion of Bohn (2001)): International capital flows are an argument against the closed economy assumption. Also, as one would expect, Bütler and Harms (2001) show, that endogenous labor supply will dampen the movement of factor returns due to demographic changes. Murphy (2001) also points out, that when looking at such long-run horizons, as we do in overlapping-generations models, one should not underestimate the degree of substitutability that may occur. Unfortunately empirical evidence does not help much to defend either position. Due to the length of a single period in such a framework, even long run data sets can only deliver about five non-overlapping observations. Obviously, any results would not be generated at a very significant level.

5 Between defined contribution and defined benefit: Optimal demographic indexation

The discussed cases of defined contribution and defined benefit wage indexation are of course only the polar cases of a continuum of possibilities on how to implement the demographic indexation of PAYG social security.

The burden of the realization of a single population growth shock can be split between the current living young and old generation in any given proportion. In order to capture this in a more general setting, I introduce ρ, defined as the proportion of how much the current old generation's benefits are adjusted in response to the population growth shock. The policy rule for a wage-indexed pension scheme with balanced budgets in every period is then given by:

$$\tau_t = \gamma[w\varepsilon_t]\frac{1+n+\rho\eta_{t-1}}{1+n+\eta_{t-1}} \quad \text{and} \quad tr_{t+1} = \gamma(1+n+\rho\eta_t)[w\varepsilon_{t+1}]. \tag{22}$$

Note that the earlier discussed cases are special cases of this more general representation, where $\rho = 1$ ($\rho = 0$) equals DC (DB). It seems reasonable to impose the restriction of $0 \leq \rho \leq 1$.

5.1 Optimal demographic indexation in the SMOPEC

Life-cycle resources for the more general demographic indexation in the SMOPEC are obtained by inserting the pension policy from equation 22 and labor income from equation 1 into equation 2:

$$\begin{aligned} y_t &= w\varepsilon_t \left[1 - \gamma \frac{1+n+\rho\eta_{t-1}}{1+n+\eta_{t-1}}\right] + \gamma w \varepsilon_{t+1} \frac{1+n+\rho\eta_t}{1+r} \\ &= w\varepsilon_t \left[1 - \gamma \left(\rho + \frac{(1-\rho)(1+n)}{1+n+\eta_{t-1}}\right)\right] + \gamma w \varepsilon_{t+1} \frac{1+n+\rho\eta_t}{1+r} \end{aligned} \tag{23}$$

The expectation of equation 23 is given by:

$$\begin{aligned} \mathrm{E}[y_t] &= w - \gamma w \left(\rho + (1-\rho)(1+n)\,\mathrm{E}\left[\frac{1}{1+n+\eta_{t-1}}\right]\right) + \gamma w \frac{1+n}{1+r} \\ &\approx w - \gamma w \frac{r-n}{1+r} - (1-\rho)\gamma w \frac{\sigma_\eta^2}{(1+n)^2} \end{aligned} \tag{24}$$

The approximate solution given in the second line of equation 24 is obtained by substituting the quadratic approximation of $(1+n+\eta_{t-1})^{-1}$ from equation 12 into the second line of equation 24. As before, moving towards a scheme with defined benefit elements ($\rho < 1$) reduces the expectation of life-cycle resources. Because $\mathrm{E}[y_t]$ is strictly increasing in ρ for $\eta > -(1+n)$ unless $\mathrm{E}[\eta] = 0$ with probability one, the expectation of life cycle resources is maximized in the corner solution of $\rho = 1$ (DC).[13]

An approximate solution for the variance of life-cycle resources is obtained by substituting the linear approximation of $(1+n+\eta_{t-1})^{-1}$ into the second line of equation 23 and using the familiar definition of the variance:

$$\begin{aligned} \mathrm{Var}^{la}[y] = w^2 \Bigg\{ & \left[(1-\gamma)^2 + \gamma^2 \left(\frac{1+n}{1+r}\right)^2\right] \sigma_\varepsilon^2 \\ & + \left[(1-\rho)^2 \left(\frac{\gamma}{1+n}\right)^2 + \rho^2 \left(\frac{\gamma}{1+r}\right)^2\right] \sigma_\eta^2 (1+\sigma_\varepsilon^2) \Bigg\}. \end{aligned} \tag{25}$$

The similarities to section 3 are obvious: The variances under DC and DB are identical in the SMOPEC except for the different "discounting mechanisms" of the terms associated with the variance of population growth. This is again the case here, with ρ determining

[13] That $\mathrm{E}[y_t]$ is strictly increasing in ρ is proofed for the general case in appendix A.4. For the approximate solution this can be easily seen by taking the partial derivative of line two in equation 24 with respect to ρ: $\frac{\partial\, \mathrm{E}[y]^{qa}}{\partial \rho} = \gamma w \frac{\sigma_\eta^2}{(1+n)^2}$.

the weights of the different "discounting mechanisms".

In the polar cases of DC and DB, the terms associated with the variance of the population growth rate are the result of the uncertainty of the realization of the shock in one of the two periods. In a hybrid scheme, where $0 < \rho < 1$, this risk can be reduced because the pension policy allows the individuals to participate in the realization of the demographic shock of both periods. This leads to a reduction of the demographic risk for the individuals in comparison to the two polar cases. The value ρ^* that minimizes the variance of life-cycle resources is given by $\rho^* = \frac{1}{1+\left(\frac{1+n}{1+r}\right)^2}$. For intergenerational risk sharing neither of the two polar cases is optimal. Instead a policy that splits the risk, that a single demographic shock has on the return of the pension program in roughly equal parts between the living generations is suited best to share the demographic risk. This bears a strong familiarity to the original results concerning intergenerational risk sharing of stochastic labor income under deterministic population growth. Note that ρ^* is independent of γ or the variances of the two random variables and will equal 0.5 for $r = n$. For dynamic efficiency ($r > n$) ρ^* will be larger than 0.5 indicating that the optimal policy is closer to DC than DB. This is in line with the result derived in section 3: For dynamic efficiency the difference in the discounting mechanism will favor DC over DB.

However, even though the demographic risk can be minimized by a pension policy that lies in between DC and DB one should still keep in mind, that in the SMOPEC the demographic risk would not be present without PAYG social security. Also note, that the splitting rule that minimizes the demographic risk will also be subject to an insurance premium, since the expectation of life-cycle resources is reduced for $\rho < 1$ in comparison to a fully funded scheme ($\gamma = 0$) or a pure DC policy ($\rho = 1$). A welfare maximizing policy will therefore depend on the degree of risk aversion of the individuals.

5.2 Optimal demographic indexation in the closed economy

As in section 4, I now consider macroeconomic effects of population growth on labor income. Labor income is assumed to behave as specified in equation 16. Substituting this and the general specification of the pension policy given in equation 22 into equation 2 yields the life-cycle resources for the closed economy:

$$\begin{aligned} y_t =& w(\varepsilon_t - \alpha\eta_{t-1}) \left[1 - \gamma \frac{1+n+\rho\eta_{t-1}}{1+n+\eta_{t-1}}\right] + \gamma w(\varepsilon_{t+1} - \alpha\eta_t) \frac{1+n+\rho\eta_t}{1+r} \\ =& w(\varepsilon_t - \alpha\eta_{t-1}) \left[1 - \gamma \left(\rho + \frac{(1-\rho)(1+n)}{1+n+\eta_{t-1}}\right)\right] \\ &+ \gamma w(\varepsilon_{t+1} - \alpha\eta_t) \frac{1+n+\rho\eta_t}{1+r} \end{aligned} \tag{26}$$

By substituting the quadratic approximation for $(1+n+\eta_{t-1})^{-1}$ into the second line of equation 26 one can derive the approximate solution of the expectation of life-cycle resources in the closed economy:

$$\mathrm{E}[y_t] \approx w - \gamma w \frac{r-n}{1+r} - (1-\rho)\gamma w \frac{\sigma_\eta^2}{(1+n)^2} - \alpha\gamma w \left(\frac{1-\rho}{1+n} + \frac{\rho}{1+r}\right)\sigma_\eta^2 \tag{27}$$

As in section 4, apart from considerations concerning dynamical efficiency or inefficiency, the expectation of life-cycle resources are reduced by PAYG social security ($\gamma > 0$). To verify this, note that the last term in equation 27 is unambiguously negative for $\gamma > 0$ and $\rho \in [0,1]$. While the expectation of life-cycle resources in the closed economy is not a strictly positive function in ρ for all parameter values, as it is the case in section 5.1, the restriction that needs to be satisfied for $\mathrm{E}[y_t]$ being maximized at $\rho = 1$, requires only that the economy is not in an extremely dynamically inefficient region ($n \gg r$).[14] So again, the expectation of life-cycle resources is maximized at $\rho = 1$ (DC) for realistic parameter values.

The approximate variance can be obtained by substituting the linear approximation of $(1+n+\eta_{t-1})^{-1}$ into the second line of equation 26 (see appendix A.5):

$$\begin{aligned}\mathrm{Var}^{la}[y_t] = w^2 \Bigg\{ & \left[(1-\gamma)^2 + \gamma^2\left(\frac{1+n}{1+r}\right)^2\right](\sigma_\varepsilon^2 + \alpha^2\sigma_\eta^2) \\ & + \left[(1-\rho)^2\left(\frac{\gamma}{1+n}\right)^2 + \rho^2\left(\frac{\gamma}{1+r}\right)^2\right]\left[\sigma_\eta^2(1+\sigma_\varepsilon^2) + 2\alpha^2\sigma_\eta^4\right] \\ & - 2\left[(1-\rho)\frac{\gamma(1-\gamma)}{1+n} + \rho(1+n)\left(\frac{\gamma}{1+r}\right)^2\right]\alpha\sigma_\eta^2\Bigg\}.\end{aligned} \tag{28}$$

Equation 28 is the general version of all other cases derived earlier. Accordingly, all other results discussed until now can be reproduced by choosing the correct parameters: $\alpha = 0$ and $\rho \in \{0,1\}$ will generate the results of section 3, $\alpha > 0$ and $\rho \in \{0,1\}$ produce the variances given in section 4 and finally $\alpha = 0$ and $\rho \in [0,1]$ yields equation 25 discussed in section 5.1. Not surprisingly, the interpretation of equation 28 draws on the different results derived in the previous sections. The first line shows the variance reducing effect of PAYG social security when labor income is subject to two kinds of risks: The risk of fluctuating labor income because of both uncertain productivity and uncertain demographic growth is reduced by sharing this risk across generations (compare section 4). The second line shows, that the risk of an uncertain return form the social security scheme that was branded the ”necessary evil” of PAYG social security from a risk perspective, can be minimized by choosing a scheme that shares the demographic shock

[14] The precise restriction is $\alpha(n-r)\frac{1+n}{1+r} < 1$.

roughly in even parts (ρ near 0.5 depending on r and n). The third line shows the "covariance effect" already discussed in section 4: Because the replacement rate and the underlying labor income that determine the transfer payment move in opposite direction, the riskiness of the return of the pension scheme is automatically reduced in the closed economy. The "covariance effect" in equation 28 is equivalent to the respective effects in the polar cases of section 4 (see equations 19 and 20) with ρ determining the weights of the the single covariance terms. Note that the variance increasing effect of line two is reduced in comparison to section 4, but the variance reducing "covariance effect" in line three is not.

The risk minimizing level for demographic indexation ρ^* in the closed economy is given by:

$$\rho^* = \frac{1}{1+\left(\frac{1+n}{1+r}\right)^2} + \alpha(1+n)\frac{1+\left(\frac{1+n}{1+r}\right)^2 - \frac{1}{\gamma}}{\left[1+\left(\frac{1+n}{1+r}\right)^2\right]\left[1+\sigma_\varepsilon^2+2\alpha^2\sigma_\eta^2\right]}. \tag{29}$$

The first term on the RHS of equation 29 is familiar from the optimal demographic indexation in the SMOPEC: Depending on r and n the variance can be minimized by choosing a level for the demographic indexation that roughly equals 0.5. In the closed economy, the second term in equation 29 is added. Three properties of ρ^* in the closed economy are noteworthy. Firstly, ρ^* is an increasing function in γ. Secondly, ρ^* should be smaller in the closed economy than in the SMOPEC if $1+(\frac{1+n}{1+r})^2 < \frac{1}{\gamma}$. For $r = n$ this condition is reduced to $\gamma < \frac{1}{2}$. Thirdly, for $1+(\frac{1+n}{1+r})^2 < \frac{1}{\gamma}$ the optimal demographic indexation, ρ^*, is a decreasing function in α for realistic parameter values.[15] All three properties of ρ^* are closely linked to the difference in the "covariance effects" of DC and DB already discussed in section 4. Because of the "covariance effect" DC provides better insurance for the η_t shock and DB provides better insurance for the η_{t-1} shock. Depending on the size of social security ($\gamma < 0.5$ for $r = n$), η_{t-1} has a larger influence on life-cycle resources than η_t and therefore DB dominates DC from a risk perspective. This is equivalent here, only that the optimal policy does not swing from one extreme to the other, but gradually adjusts towards one of the two schemes. The third property points out, that for larger α, i.e. a stronger influence of demographics on the wage rate, this "covariance effect" will have a stronger influence on the optimal choice of ρ.

[15]The exact conditions for ρ^* decreasing in α are: $1+(\frac{1+n}{1+r})^2 < \frac{1}{\gamma}$ and $\alpha < \sqrt{\frac{1+\sigma_\varepsilon^2}{2\sigma_\eta^2}}$.

6 Conclusion

Expanding the analysis of Thøgersen (1998) by an uncertain population growth rate may put the intergenerational risk sharing features of wage-indexed pension programs into perspective. In a small open economy the reduction of labor income risk via these programs is bought at the expense of an added demographic risk that is not present in a fully funded system.

The inclusion of the stochastic population growth rate makes it necessary to specify how the realization of the demographic random variable will affect contributions and benefit payments of the pension program. In sections 3 and 4, the policy options are restricted to the two extreme cases: Defined contribution wage indexation and defined benefit wage indexation. The essential difference between the two schemes is the exposure to the demographic shock. Under defined contribution the risk of the demographic shock is borne by the old generation, whereas under defined benefit the young generation's contribution payments are adjusted in response to the realization of the population growth rate. In section 5, a more general policy concerning the demographic indexation of PAYG social security is introduced: The impact of the population growth rate on the pension program can be split in any proportion between the currently living young and old generation.

For the two polar cases, two general distinctions are shown: Firstly, the expectation of life-cycle resources is always greater under DC than under DB. The reason for this is, that life-cycle resources are a linear function of the demographic random variable under the former, but a concave function of that variable under the latter. Secondly, due to the different timing of the transmission of the demographic shock within the two schemes, the discounting mechanism for the component that is associated with the variance of population growth is different: While under DC it is discounted by the gross interest rate, it is discounted by the gross population growth rate under DB. These two differences between DC and DB are not dependent on whether the economy is closed or open, but will hold in general.

A comparison of DC versus DB in a small open economy (section 3) has shown, that if the economy is dynamically efficient, DC tends to share risks better than DB if the level of social security is not too low. This is due to the different discounting mechanisms of the demographic shock within the two schemes. Also, the expectation of life-cycle resources is larger under DC than under DB. This leads to the conclusion that in a SMOPEC from an ex-ante perspective a defined contribution scheme is to be preferred to a defined benefit scheme for $r > n$ (dynamic efficiency).

In section 4, I allow for a macroeconomic feedback effect of population growth on labor income in order to mimic general equilibrium effects of a closed economy. The results between section 3 and section 4 differ significantly. Firstly, in a SMOPEC life-cycle resources are not subject to demographic risks without a PAYG pension program.

Gains from sharing labor income riskbetween generations via social security are therefore reduced because a demographic risk must be added. In a closed economy, this is very different: Labor income is subject to demographic changes itself and the risk of uncertain labor income due to fertility shocks is also reduced via social security. However, social security still adds uncertainty to life-cycle resources because of the uncertain return from the pension scheme. So depending on the size of the macroeconomic feedback effect, demographic risks are actually reduced by social security in the closed economy instead of increased, as it is the case in the SMOPEC. However, this insurance against the influence of demographic shocks on labor income is not for free: Under both policies the expectation of life-cycle resources is reduced because of the demographic uncertainty. This reduction is independent of dynamic efficiency or dynamic inefficiency and therefore comes in addition to the well known results concerning the return of PAYG social security (see Aaron (1966)).

The second distinction between the SMOPEC and the closed economy concerns the specific design of wage-indexed social security: In the closed economy life-cycle resources are touched by the realizations of the two demographic shocks, η_{t-1} and η_t, in three ways. Firstly, η_{t-1} affects generation's t own labor income. Secondly, η_t affects the labor income of generation $t+1$, which, for a wage-indexed pension scheme, affects the retirement payments of generation t. And thirdly, depending on the policy, either the replacement rate is adjusted in response to the shock η_t (DC) or the contribution rate is adjusted in response to η_{t-1} (DB). The respective third effect will always be of different direction than the first (DB) or the second effect (DC). The combination of the third and first effect or third and second effect, respectively, can then be interpreted as a "covariance" between the respective basis and the applicable contribution rate (DB) or replacement rate (DC). Since this "covariance" is negative, it provides additional insurance against movements in labor income due to demographic changes. The difference between DC and DB is, that DB provides this additional insurance for the shock η_{t-1}, while DC does so for n_t. So if a generation's own labor income has the largest weight in that generation's life-cycle resources, what can generally be assumed, DB provides better insurance than DC.

In section 5, a more general policy specification concerning the demographic indexation of the pension program is introduced: The influence of the demographic shock of a single period on the payments to and from the social security scheme can be split in any given proportion. Under this general specification, the risk, that is added to life-cycle resources by PAYG social security because of the uncertain return on the contributions paid into the scheme, can be significantly reduced by choosing the correct demographic indexation. Specifically, a policy that splits the financial effect of the demographic random variable on social security roughly equally between the living generations is suited best to reduce the variance of life-cycle resources. Also, the stark difference between the SMOPEC and the closed economy concerning the optimal choice of DC and DB is no longer present. Depending on various parameters the optimal demographic indexation only gradually moves away from the "split-evenly" rule.

A Appendix

A.1 Derivation of the variance of y_t^{DC}

The variance of life-cycle resources under DC in the closed economy can be calculated using y^{DC} and $\mathrm{E}[y^{DC}]$ from equations 17 and 18, respectively. By definition the variance is then:

$$\begin{aligned}\mathrm{Var}[y_t^{DC}] =& \mathrm{E}\left[\left(y_t^{DC}\right)^2\right] - \left(\mathrm{E}[y_t^{DC}]\right)^2 \\ =& \mathrm{E}\left[\left(w(\varepsilon_t - \alpha\eta_{t-1})(1-\gamma) + \frac{\gamma w}{1+r}(\varepsilon_{t+1} - \alpha\eta_t)(1+n+\eta_t)\right)^2\right] \\ & - \left(w(1-\gamma) + \gamma w\frac{1+n}{1+r} - \alpha w\frac{\gamma}{1+r}\sigma_\eta^2\right)^2.\end{aligned}$$

For multiplying the quadratic terms one has to keep in mind, that all random variables $(\varepsilon_t, \varepsilon_{t+1}, \eta_t, \eta_{t-1})$ are assumed to be independent of each other. Among others, the following moments are used: $\mathrm{E}[\varepsilon_{t+1}^2\eta_t^2] = \mathrm{E}[\varepsilon_{t+1}^2]\,\mathrm{E}[\eta_t^2] = (\sigma_\varepsilon^2+1)\sigma_\eta^2$, $\mathrm{E}[\eta_t^4] = 3\sigma_\eta^4$.

Also note that all odd moments of η are zero. This is due to the assumption of η being normally distributed with mean zero. For all symmetric distributions, which applies for the normal, all odd central moments are zero. Since η is mean zero, all moments of η are equal to the respective central moments.

After multiplying, employing the expectations operator and canceling terms we get the variance given in equation 19. Setting α equal to zero yields the variance of y^{DC} for the small open economy given in equation 8.

A.2 Derivation of the variance of y_t^{DB} in the SMOPEC

I first derive the variance of y^{DB} for the SMOPEC without macroeconomic feedback ($\alpha = 0$). From the definition of the variance we have:

$$\begin{aligned}\mathrm{Var}[y^{DB}] =& \mathrm{E}\left[\left(y_t^{DB}\right)^2\right] - \left(\mathrm{E}[y_t^{DB}]\right)^2 \\ =& \mathrm{E}\left[\left(w\varepsilon_t\left(1 - \frac{\psi}{1+n+\eta_{t-1}}\right) + w\varepsilon_{t+1}\frac{\psi}{1+r}\right)^2\right] \\ & - \left(w + \frac{w\psi}{1+r} - w\psi\,\mathrm{E}\left[\frac{1}{1+n+\eta_{t-1}}\right]\right)^2.\end{aligned}$$

Using the quadratic approximation for $(1 + n + \eta)^{-1}$ given in equation 12 yields the quadratic approximation of the variance of y^{DB}:

$$\begin{aligned}\mathrm{Var}^{qa}[y^{DB}] =& w^2\,\mathrm{E}\Bigg[\bigg\{\varepsilon_t\left(1-\psi\left(\frac{1}{1+n}-\frac{\eta_{t-1}}{(1+n)^2}+\frac{\eta_{t-1}^2}{(1+n)^3}\right)\right)\\ &+\varepsilon_{t+1}\frac{\psi}{1+r}\bigg\}^2\Bigg]\\ &-w^2\left(\left(1-\frac{\psi}{1+n}\right)+\frac{\psi}{1+r}-\psi\left(\frac{1}{1+n}\right)^3\sigma_\eta^2\right)^2.\end{aligned} \tag{30}$$

After having substituted the quadratic approximation for $(1+n+\eta_{t-1})^{-1}$ one only needs to take expectations over simple moments that can be derived by the moment generating function for the normal distribution. The quadratic approximation of the variance under DB then equals:

$$\begin{aligned}\mathrm{Var}^{qa}[y^{DB}] =& w^2\Bigg\{\left[\left(1-\frac{\psi}{1+n}\right)^2+\left(\frac{\psi}{1+r}\right)^2\right]\sigma_\varepsilon^2+\frac{\psi^2}{(1+n)^4}(1+\sigma_\varepsilon^2)\sigma_\eta^2\\ &-2\frac{\psi}{(1+n)^3}\left(1-\frac{\psi}{1+n}\right)\sigma_\eta^2\sigma_\varepsilon^2+\psi^2\left(\frac{1}{1+n}\right)^6(2\sigma_\eta^2+3\sigma_\eta^2\sigma_\varepsilon^2)\Bigg\}.\end{aligned} \tag{31}$$

Deriving the linear approximation of this variance will be easier: When substituting for $(1 + n + \eta_{t-1})^{-1}$ the last term of the quadratic approximation can be neglected. The linear approximation is equal to the first line in equation 31.

A.3 The variance of y_t^{DB} in a closed economy using the quadratic approximation

Substituting the quadratic approximation for $(1 + n + \eta_{t-1})^{-1}$ given in equation 12 into y_t^{DB} given in equation 17 yields the following variance:

$$\begin{aligned}\mathrm{Var}^{qa}[y^{DB}] =& w^2\Bigg\{\left[\left(1-\frac{\psi}{1+n}\right)^2+\left(\frac{\psi}{1+r}\right)^2\right](\sigma_\varepsilon^2+\alpha^2\sigma_\eta^2)\\ &+\frac{\psi^2}{(1+n)^4}(1+\sigma_\varepsilon^2)\sigma_\eta^2\\ &+2\frac{\psi^2}{(1+n)^4}\alpha^2\sigma_\eta^4-2\frac{\psi}{(1+n)^2}\left(1-\frac{\psi}{1+n}\right)\alpha\sigma_\eta^2\\ &-2\frac{\psi}{(1+n)^3}\left(1-\frac{\psi}{1+n}\right)\sigma_\eta^2\sigma_\varepsilon^2+\frac{\psi^2}{(1+n)^6}(2\sigma_\eta^2+3\sigma_\eta^2\sigma_\varepsilon^2)\\ &-6\frac{\psi}{(1+n)^3}\left(1-\frac{\psi}{1+n}\right)\alpha^2\sigma_\eta^4+10\frac{\psi^2}{(1+n)^5}\alpha\sigma_\eta^2\Bigg\}.\end{aligned} \tag{32}$$

A.4 Proof that $\mathrm{E}[y_t]$ is strictly increasing in ρ

Taking the partial derivative of the expectation of life-cycle resources given on the RHS on line one of equation 24 with respect to ρ yields:

$$\frac{\partial\,\mathrm{E}[y_t]}{\partial\rho} = \gamma w\left((1+n)\,\mathrm{E}\left[\frac{1}{1+n+\eta_{t-1}}\right]-1\right) \tag{33}$$

Because $(1+n+\eta_{t-1})^{-1}$ is strict convex for $\eta_{t-1} < -(1+n)$ we have from Jensen's inequality that $\mathrm{E}\left[\frac{1}{1+n+\eta_{t-1}}\right] > \frac{1}{1+n+\mathrm{E}[nt]}$ unless $\mathrm{E}[\eta_{t-1}]=0$ with probability one. This implies that:

$$(1+n)\,\mathrm{E}\left[\frac{1}{1+n+\eta_{t-1}}\right] > \frac{1+n}{1+n+\mathrm{E}[\eta_{t-1}]} = 1. \tag{34}$$

From equations 33 and 34 one can easily see that $\frac{\partial\,\mathrm{E}[y_t]}{\partial\rho} > 0$.

A.5 Derivation of the variance under the general demographic indexation policy in the closed economy

Substituting the linear approximation of $(1+n+\eta_{t-1})^{-1}$ into the second line of equation 26 yields:

$$\begin{aligned} y_t \approx & w(\varepsilon_t-\alpha\eta_{t-1})\left[1-\gamma\left(1-\frac{1-\rho}{1+n}\eta_{t-1}\right)\right] \\ & +\gamma w(\varepsilon_{t+1}-\alpha\eta_t)\frac{1+n+\rho\eta_t}{1+r} \end{aligned} \tag{35}$$

The expectation of equation 35 is:

$$\mathrm{E}^{la}[y_t] = w(1-\gamma)+\gamma w\frac{1+n}{1+r}-\alpha\gamma w\left(\frac{1-\rho}{1+n}+\frac{\rho}{1+r}\right)\sigma_\eta^2,$$

so that the variance given in equation 28 can be derived by solving:

$$\begin{aligned} \mathrm{Var}^{la}[y_t] = w^2\Bigg\{ & \mathrm{E}\Bigg[\Bigg((\varepsilon_t-\alpha\eta_{t-1})\left[1-\gamma\left(1-\frac{1-\rho}{1+n}\eta_{t-1}\right)\right] \\ & +\gamma(\varepsilon_{t+1}-\alpha\eta_t)\frac{1+n+\rho\eta_t}{1+r}\Bigg)^2\Bigg] \\ & -\left((1-\gamma)+\gamma\frac{1+n}{1+r}-\alpha\gamma\left(\frac{1-\rho}{1+n}+\frac{\rho}{1+r}\right)\sigma_\eta^2\right)^2\Bigg\}. \end{aligned} \tag{36}$$

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